

Course material

Course:

Digital Signal Processing II

Video:

2.1.4.b Examples of frequency response

Concepts (extracted from automatically generated subtitles):

Frequency response. Smooth signal. Magnitude of the dtft. Time domain. Fourier transform of the noise. Zero phase. Times m. Effects of the phase. Phase term. Carpler-strong algorithm. Minus lambda times lambda. Impulse response. Frequency axis. Spectrum of the smooth signal. Fourier transforms.



[to video sequence search](#)
(within Digital Signal Processing II.)



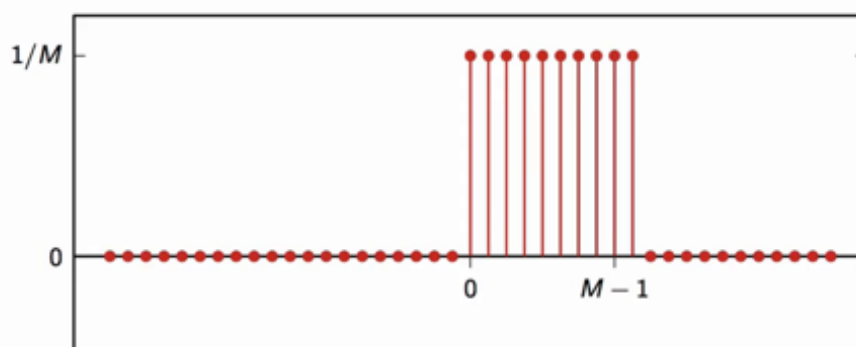
[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

Moving Average revisited

$$h[n] = (u[n] - u[n - M])/M$$



Let's go back now to our friend the moving average and compute its frequency response. The moving average has an impulse response which is just the indicator sequence for the interval 0 to M-1 divided by M. We have already computed the DTFT of such a signal a few lessons back.

notes

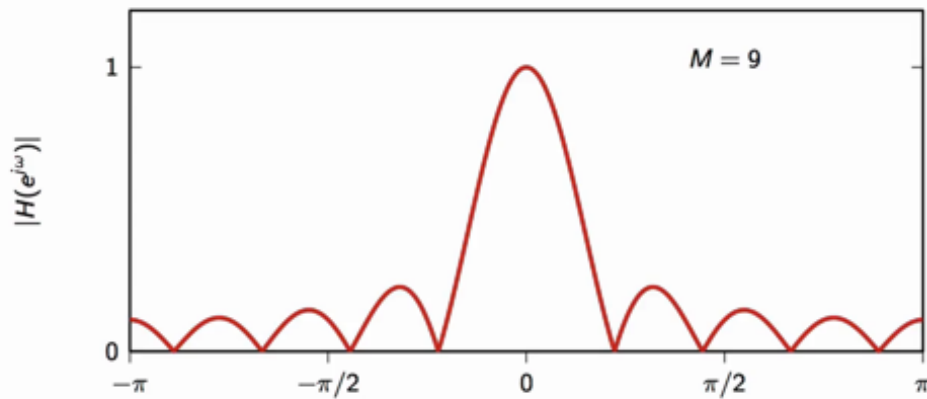
summary

0m 0s



Moving Average, magnitude response

$$|H(e^{j\omega})| = \frac{1}{M} \left| \frac{\sin(\frac{\omega}{2} M)}{\sin(\frac{\omega}{2})} \right|$$



If we consider now the magnitude of the DTFT we find out that it's just 1 over capital M multiplies the absolute value of sine of omega over 2 times M divided by sine of omega over 2. An interesting thing to remember is that the magnitude response is going to be 0 at multiples of 2 pi over capital M except in 0.

notes

summary

0m 16s

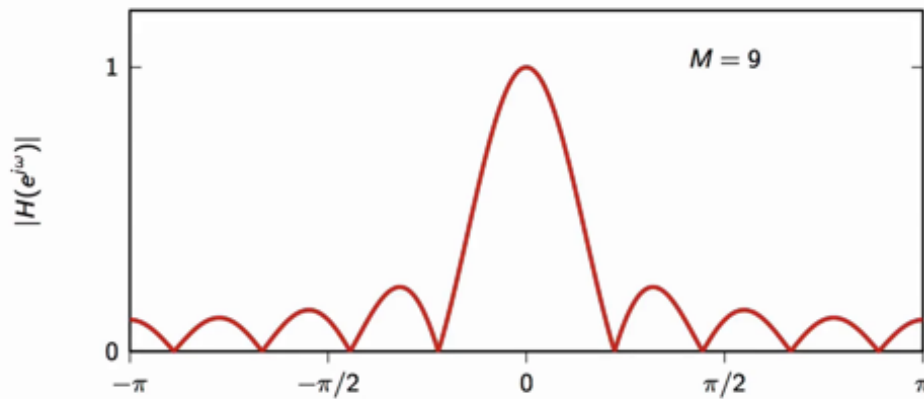


Moving Average, magnitude response

$$|H(e^{j\omega})| = \frac{1}{M} \left| \frac{\sin(\frac{\omega}{2} M)}{\sin(\frac{\omega}{2})} \right|$$

$$\frac{2\pi}{M} k$$

$k \neq \dots$



So here you see that you have indeed capital M minus 1 zeros along the frequency axis.

notes

summary

0m 36s

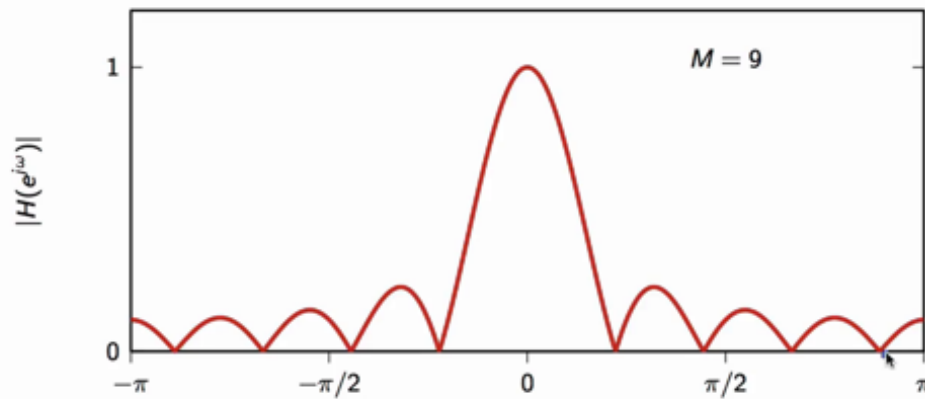


Moving Average, magnitude response

$$|H(e^{j\omega})| = \frac{1}{M} \left| \frac{\sin(\frac{\omega}{2} M)}{\sin(\frac{\omega}{2})} \right|$$

$$\frac{2\pi}{M} k$$

$k \neq 0$



We can plot the magnitude response for increasing values of capital M.

notes

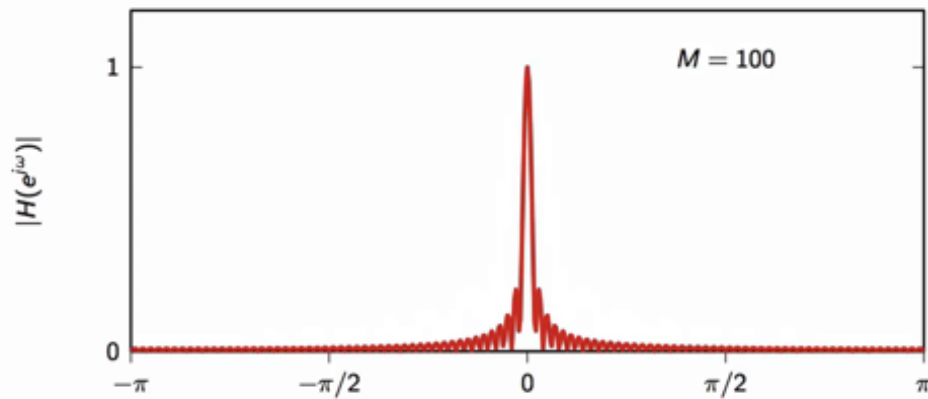
summary

0m 48s



Moving Average, magnitude response

$$|H(e^{j\omega})| = \frac{1}{M} \left| \frac{\sin(\frac{\omega}{2} M)}{\sin(\frac{\omega}{2})} \right|$$



So for instance here you have the frequency response for M equal to 20 and you could count the 19 zeros along the frequency axis. And here you have the frequency response for capital M equal to 100 and you can see that the frequency response is very concentrated around the origin. We will see the significance of that later on.

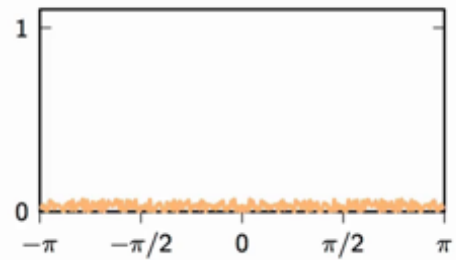
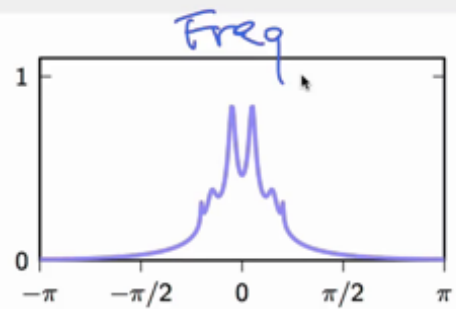
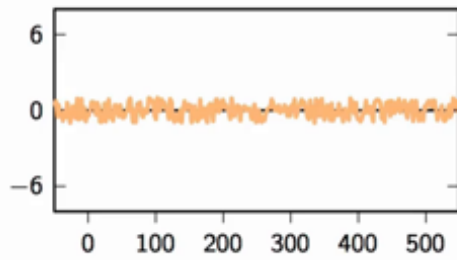
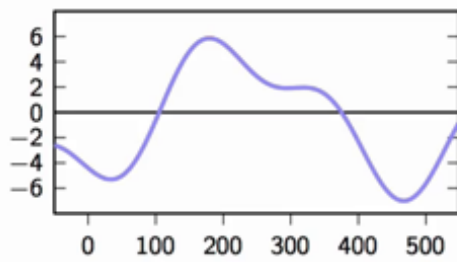
notes

summary

0m 49s



Denoising revisited



Let's go back to our denoising example that we studied in the time domain in the previous module and look at its development in the frequency domain. So remember we had a smooth signal, here we are talking time domain, so we had a smooth signal that was corrupted by some additive noise that we represent here in orange. In the frequency domain the spectrum of the smooth signal looks like this.

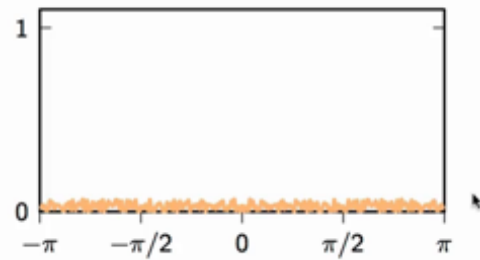
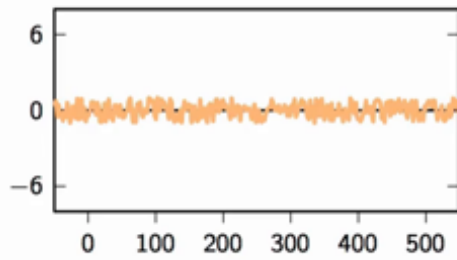
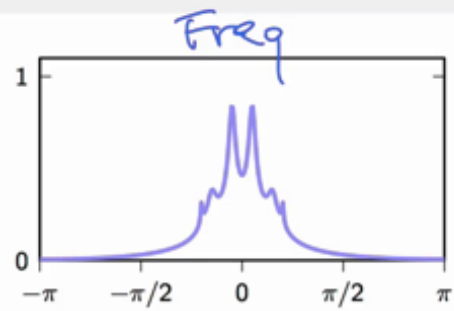
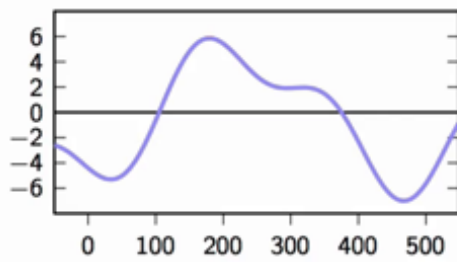
notes

summary

1m 25s



Denoising revisited



Most of its energy is concentrated around the low frequency so it's a low pass signal.
And the Fourier transform of the noise looks like noise in the frequency domain as well.

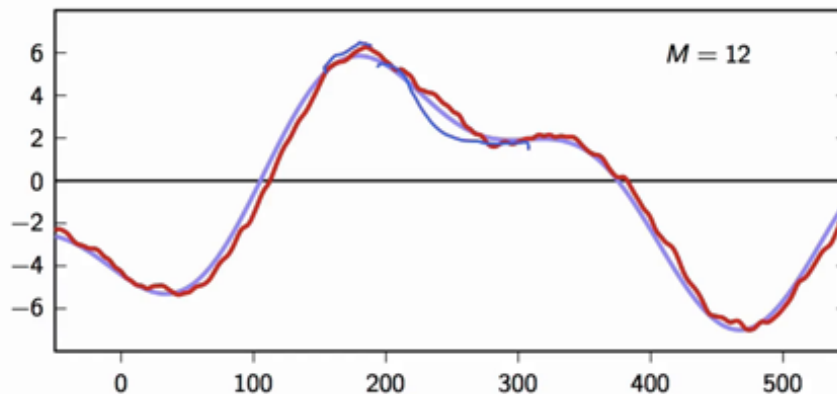
notes

summary

1m 56s



By the way, remember the time-domain analysis...



So when we put things together we have the spectrum of the smooth signal, we have the spectrum of the noise and the sum is the noisy spectrum of the measured signal. When we use a moving average to filter this noisy signal in the frequency domain we're multiplying this spectrum by the frequency response of the moving average. And in magnitude it looks like this. So here we're using for instance a 9 point average as you can see from the number of zeros along the frequency axis. So the product in magnitude determines a very deep attenuation of the high frequency and therefore most of the noise that was contained in these bands has been eliminated. At the same time though if you compare the result of the filtering operation with the original spectrum you see that the filtering has eliminated parts of the original spectrum that actually had every right to be there. So that proves that there's no free lunch and signal processing as well and in order to remove the noise sometimes you have to remove some of the good part with it as well. Before we move on let's look once again at the same denoising operation in the time domain now that we know what's going on in the frequency domain. And we can see that when we use a moving average of say length 12 we are indeed smoothing out the highest components of the noise but not so much the ones that are slower moving. And that we are already starting to alter a little bit the curvature of the signal as explained by the spectrum we just derived. So what about the phase? The best way to understand the

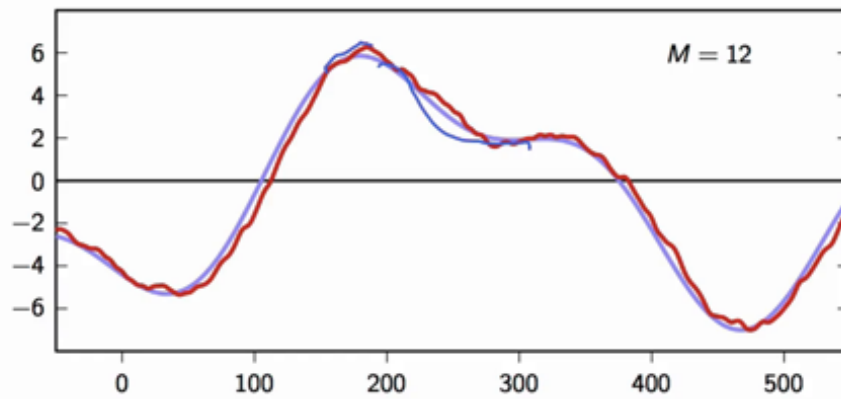
notes

summary

2m 7s



By the way, remember the time-domain analysis...



effects of the phase on a signal is to distinguish three different cases.

notes

summary

What about the phase?

Assume $|H(e^{j\omega})| = 1$

- ▶ zero phase: $\angle H(e^{j\omega}) = 0$
- ▶ linear phase: $\angle H(e^{j\omega}) = d\omega$
- ▶ nonlinear phase

The first one is zero phase which means the spectrum is real. The second case is linear phase where the phase is proportional to the frequency via a real factor D .

notes

summary

3m 42s



What about the phase?

Assume $|H(e^{j\omega})| = 1$

- ▶ zero phase: $\angle H(e^{j\omega}) = 0$
- ▶ linear phase: $\angle H(e^{j\omega}) = d\omega$
- ▶ nonlinear phase

And the third case is nonlinear phase which covers all other possibilities. To understand what phase does to a signal let's take a very simple discrete time signal made up of the sum of two sinusoids.

notes

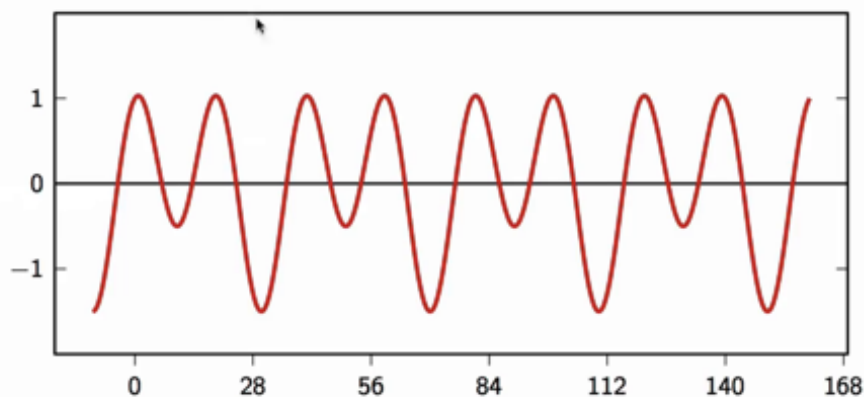
summary

3m 54s



Phase and signal shape

$$x[n] = \frac{1}{2} \sin(\omega_0 n) + \cos(2\omega_0 n) \quad \omega_0 = \frac{2\pi}{40}$$



X of n is equal to one half sine of omega zero times n plus cosine of two omega zero times n.

notes

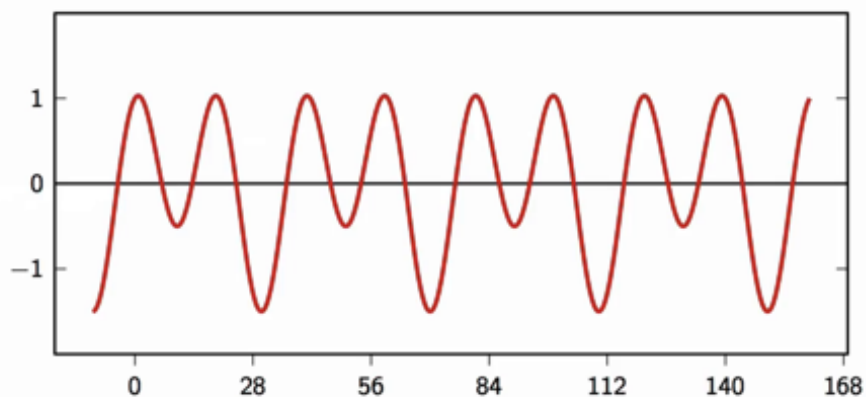
summary

4m 8s



Phase and signal shape

$$x[n] = \frac{1}{2} \sin(\omega_0 n) + \cos(2\omega_0 n) \quad \omega_0 = \frac{2\pi}{40}$$



So it's two sinusoids at frequencies one double of the other.

notes

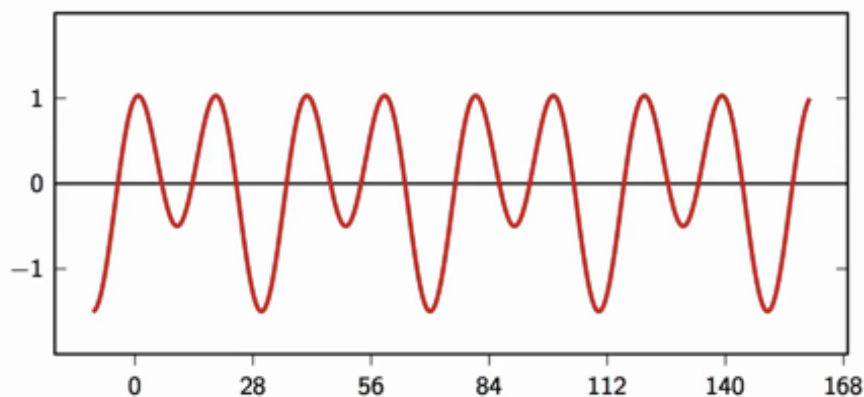
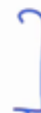
summary

4m 17s



Phase and signal shape

$$x[n] = \frac{1}{2} \sin(\omega_0 n) + \cos(2\omega_0 n) \quad \omega_0 = \frac{2\pi}{40}$$



And the sum looks like this signal that we plot here in this graph. We can call this signal a zero phase signal because the phase associated to each sinusoidal function is zero.

notes

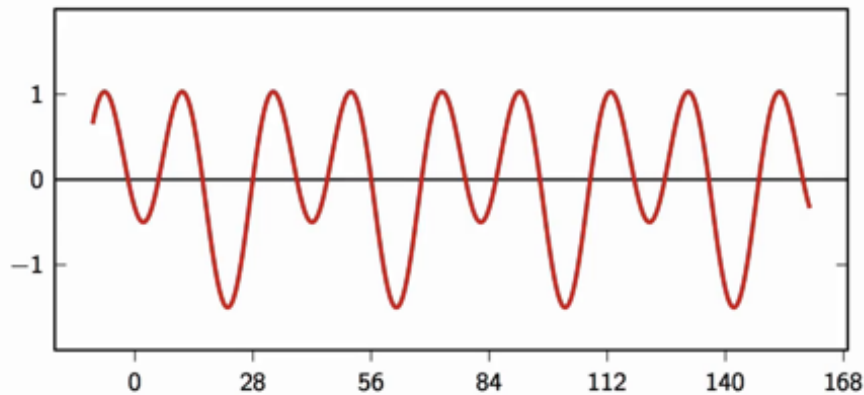
summary

4m 25s



Phase and signal shape: linear phase

$$x[n] = \frac{1}{2} \sin(\omega_0 n + \theta_0) + \cos(2\omega_0 n + 2\theta_0) \quad \theta_0 = \frac{8\pi}{5}$$



Now let's add a phase term to each component and make sure that the phase is proportional to the frequency of the sinusoid. So what we add is θ_0 to the first component at frequency ω_0 . And we add $2\theta_0$ to the second component whose frequency is $2\omega_0$. The value of θ_0 is actually irrelevant. But we can see that when we add a phase term to the sinusoidal components that is proportional to the frequency of the sinusoid,

notes

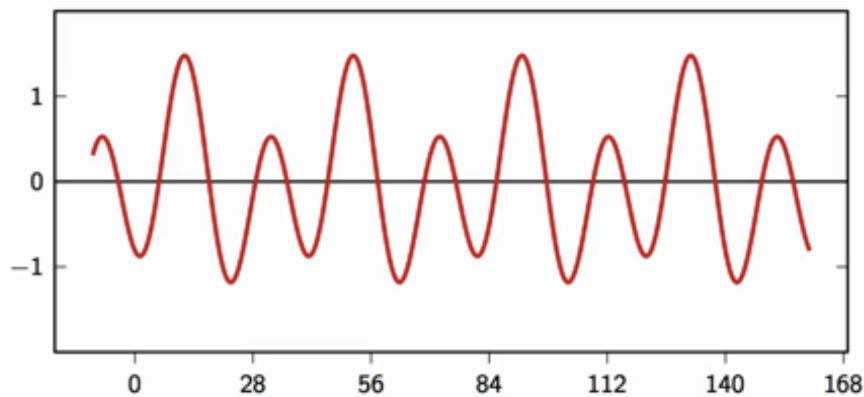
summary

4m 39s



Phase and signal shape: nonlinear phase

$$x[n] = \frac{1}{2} \sin(\omega_0 n) + \cos(2\omega_0 n + 2\theta_0)$$



the net effect in the time domain is just a shift of the signal. The shape of the signal remains exactly the same. If now we add a phase that is non-proportional to the frequency, so in this case we change the phase to the first term to zero and we leave a phase term of two theta zero to the second term, while the frequencies of the two components have not changed, the shape of the signal in the time domain has changed significantly. Please note that in all three cases the spectrum of X of n remains exactly the same in magnitude.

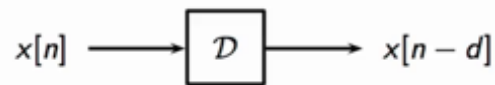
notes

summary

5m 13s



Linear phase



- ▶ $y[n] = x[n - d]$
- ▶ $Y(e^{j\omega}) = e^{-j\omega d} X(e^{j\omega})$
- ▶ $H(e^{j\omega}) = e^{-j\omega d}$
- ▶ linear phase term

To understand that the linear phase term is simply a delay in the time domain, consider the following system where we have a filter D that simply produces a delayed version of its input. The input-output relationship is this one here. And in the frequency domain we can take Fourier transforms left and right, and we obtain that big Y of e to the j omega is equal to e to the minus j omega d times big X of e to the j omega.

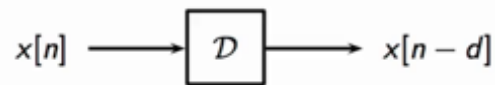
notes

summary

5m 53s



Linear phase



- ▶ $y[n] = x[n - d]$
- ▶ $Y(e^{j\omega}) = e^{-j\omega d} X(e^{j\omega})$
- ▶ $H(e^{j\omega}) = e^{-j\omega d}$
- ▶ linear phase term

This is really equivalent to saying that the frequency response of this delaying filter is simply $e^{-j\omega d}$.

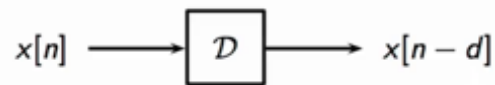
notes

summary

6m 13s



Linear phase



- ▶ $y[n] = x[n - d]$
- ▶ $Y(e^{j\omega}) = e^{-j\omega d} X(e^{j\omega})$
- ▶ $H(e^{j\omega}) = e^{-j\omega d}$
- ▶ linear phase term

So again we have a linear phase term where the phase introduced by the filter is proportional to the frequency via a factor D , which actually represents the delay in time domain. In general, if we can split the frequency response of a filter into the product of a pure real term and a pure phase term,

notes

summary

6m 27s



Linear phase

In general, if $H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{amplitude}} \underbrace{e^{-j\omega d}}_{\text{phase}}$, with $A(e^{j\omega}) \in \mathbb{R}$



it means that the filter operates by combining the action of a zero phase and therefore zero delay component

notes

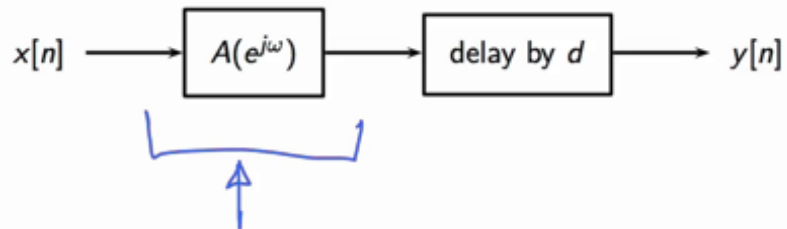
summary

6m 43s



Linear phase

In general, if $H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{magnitude}} \underbrace{e^{-j\omega d}}_{\text{phase}}$, with $A(e^{j\omega}) \in \mathbb{R}$



that only affects the magnitude of the input followed by a delay by these samples.

notes

summary

6m 54s



Moving Average is linear phase

$$H(e^{j\omega}) = \frac{1}{M} \frac{\sin(\frac{\omega}{2}M)}{\sin(\frac{\omega}{2})} e^{-j\frac{M-1}{2}\omega}$$

And this delay is inevitable in the case of causal filters for instance. Let's consider the moving average again. The frequency response is the product of a real term here and a pure phase term here,

notes

summary

7m 1s



Moving Average is linear phase

$$H(e^{j\omega}) = \frac{1}{M} \frac{\sin(\frac{\omega}{2}M)}{\sin(\frac{\omega}{2})} e^{-j\frac{M-1}{2}\omega}$$


where the delay D is exactly m minus 1 over 2,

notes

summary

7m 13s



Moving Average is linear phase

$$H(e^{j\omega}) = \frac{1}{M} \frac{\sin(\frac{\omega}{2}M)}{\sin(\frac{\omega}{2})} e^{-j\frac{M-1}{2}\omega}$$

Handwritten notes: $d = \frac{M-1}{2}$ with an arrow pointing to the phase term, and a bracket under the magnitude term.

which represents half the length of the support of the impulse response.

notes

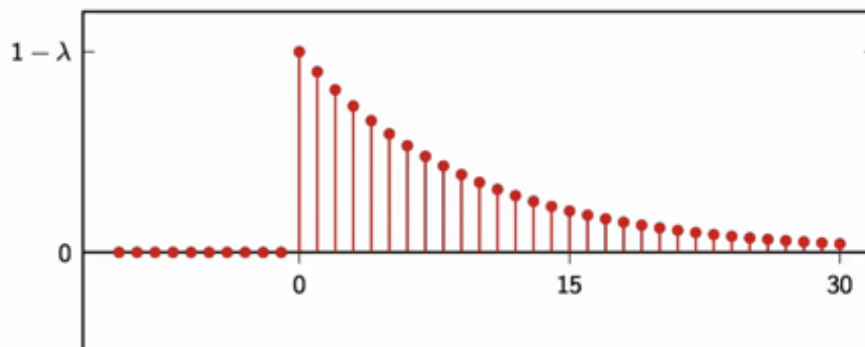
summary

7m 16s



Leaky integrator revisited

$$h[n] = (1 - \lambda)\lambda^n u[n]$$



And this is indeed the delay introduced by the moving average. So we've looked at the moving average and by now you know that what comes next is the leaky integrator. And indeed let's consider the impulse response, which you remember is 1 minus lambda times lambda to the power of n multiplied by unistep, so an exponentially decaying sequence. If we take the Fourier transform of that, we've done that in detail before,

notes

summary

7m 26s



Leaky integrator revisited

$$H(e^{j\omega}) = \frac{1 - \lambda}{1 - \lambda e^{j\omega}}$$

Finding magnitude and phase require a little algebra...

and here is the result, big H of e to the j omega is 1 minus lambda divided by 1 minus lambda times e to the j omega. Now to find the magnitude and the phase of this animal, we need to use a little algebra, and we need to recall this very simple result.

notes

summary

7m 53s



Leaky integrator revisited

Recall from complex algebra:

$$\frac{1}{a + jb} = \frac{a - jb}{a^2 + b^2}$$

so that if $x = 1/(a + jb)$,

$$|x|^2 = \frac{1}{a^2 + b^2}$$

$$\angle x = \tan^{-1} \left[-\frac{b}{a} \right]$$

If you have 1 over a plus jb, you can always rewrite that as a minus jb divided by a square plus b square.

notes

summary

8m 12s



Leaky integrator revisited

Recall from complex algebra:

$$\frac{1}{a + jb} = \frac{a - jb}{a^2 + b^2}$$

so that if $x = 1/(a + jb)$,

$$|x|^2 = \frac{1}{a^2 + b^2}$$

$$\angle x = \tan^{-1} \left[-\frac{b}{a} \right]$$

So if the number x is actually 1 over a plus jb , the magnitude square of x is 1 over a square plus b square,

notes

summary

8m 21s



Leaky integrator revisited

Recall from complex algebra:

$$\frac{1}{a + jb} = \frac{a - jb}{a^2 + b^2}$$

so that if $x = 1/(a + jb)$,

$$|x|^2 = \frac{1}{a^2 + b^2}$$

$$\angle x = \tan^{-1} \left[-\frac{b}{a} \right]$$

and the phase of x is the inverse tangent of minus b over a .

notes

summary

8m 31s



Leaky integrator revisited

$$H(e^{j\omega}) = \frac{1 - \lambda}{(1 - \lambda \cos \omega) - j \sin \omega}$$

so that:

$$|H(e^{j\omega})|^2 = \frac{(1 - \lambda)^2}{1 - 2\lambda \cos \omega + \lambda^2}$$

$$\angle H(e^{j\omega}) = \tan^{-1} \left[\frac{\lambda \sin \omega}{1 - \lambda \cos \omega} \right]$$

This applies to the leaky integrator in the sense that we can rewrite the frequency response as 1 minus lambda divided by 1 minus lambda cosine of omega, which is the real part of the denominator,

notes

summary

8m 38s



Leaky integrator revisited

so that:

$$H(e^{j\omega}) = \frac{1 - \lambda}{\underbrace{(1 - \lambda \cos \omega)}_{\text{Re}} - \underbrace{j \sin \omega}_{\text{Im}}}$$

$$|H(e^{j\omega})|^2 = \frac{(1 - \lambda)^2}{1 - 2\lambda \cos \omega + \lambda^2}$$

$$\angle H(e^{j\omega}) = \tan^{-1} \left[\frac{\lambda \sin \omega}{1 - \lambda \cos \omega} \right]$$

minus j times sine of omega, which is the imaginary part of the denominator.

notes

summary

8m 49s



Leaky integrator revisited

so that:

$$H(e^{j\omega}) = \frac{1 - \lambda}{\underbrace{(1 - \lambda \cos \omega)}_{\text{Re}} - \underbrace{j \sin \omega}_{\text{Im}}}$$

$$|H(e^{j\omega})|^2 = \frac{(1 - \lambda)^2}{1 - 2\lambda \cos \omega + \lambda^2}$$

$$\angle H(e^{j\omega}) = \tan^{-1} \left[\frac{\lambda \sin \omega}{1 - \lambda \cos \omega} \right]$$

And by applying the formula that we just saw, we find out that the magnitude square of the leaky integrator is 1 minus lambda squared

notes

summary

9m 1s



Leaky integrator revisited

so that:

$$H(e^{j\omega}) = \frac{1 - \lambda}{\underbrace{(1 - \lambda \cos \omega)}_{\text{Re}} - \underbrace{j \sin \omega}_{\text{Im}}}$$

$$|H(e^{j\omega})|^2 = \frac{(1 - \lambda)^2}{1 - 2\lambda \cos \omega + \lambda^2}$$

$$\angle H(e^{j\omega}) = \tan^{-1} \left[\frac{\lambda \sin \omega}{1 - \lambda \cos \omega} \right]$$

divided by 1 minus 2 lambda cosine of omega plus lambda square,

notes

summary

9m 9s



Leaky integrator revisited

so that:

$$H(e^{j\omega}) = \frac{1 - \lambda}{\underbrace{(1 - \lambda \cos \omega)}_{\text{Re}} - \underbrace{j \sin \omega}_{\text{Im}}}$$

$$|H(e^{j\omega})|^2 = \frac{(1 - \lambda)^2}{1 - 2\lambda \cos \omega + \lambda^2}$$

$$\angle H(e^{j\omega}) = \tan^{-1} \left[\frac{\lambda \sin \omega}{1 - \lambda \cos \omega} \right]$$

and the phase of the leaky integrator is the inverse tangent of lambda sine of omega divided by 1 minus lambda cosine of omega. So you can see that the phase is definitely nonlinear in this case.

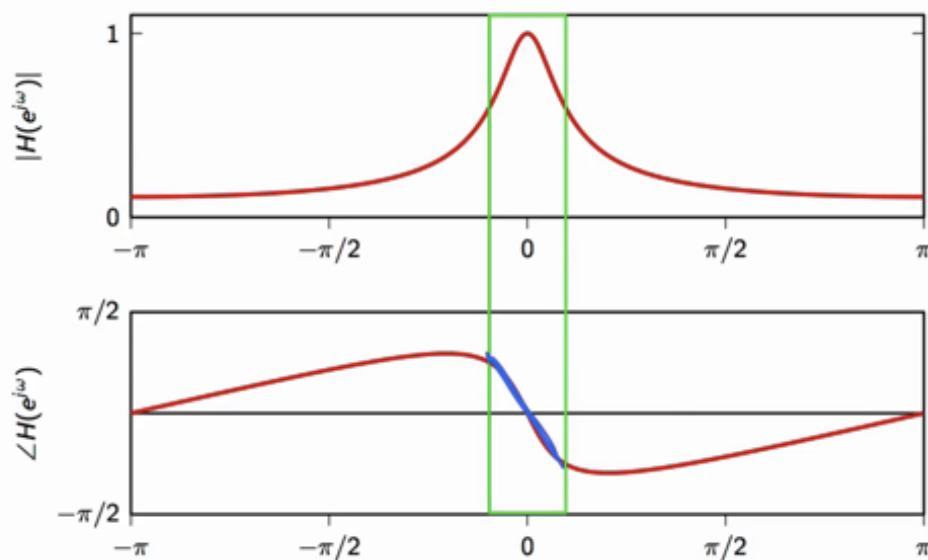
notes

summary

9m 17s



Phase is sufficiently linear where it matters



If we plot the magnitude, we can see that we have a characteristic that is very similar to that of the moving average, although we don't have any zeros, and the characteristic is monotonic rather than oscillatory. As λ goes closer to 1, we see that once again the frequency response concentrates around the origin, and for λ very, very close to 1, we have something that resembles the frequency response of a moving average computed over a large number of times. The phase looks like this. It's a nonlinear characteristic, and as λ changes, the steepness of the transition between positive and negative phase increases as well. What is interesting in terms of the combination between the magnitude and the phase response is that what interests us is the part of the filter where the attenuation is not too big, because that's where the frequencies will pass through, whereas here they will be fundamentally killed. And in that area where the frequency response has a magnitude that is sufficiently close to 1, the phase is actually more or less linear, so we can use the leaky integrator without incurring excessive phase distortion in the past man. Finally, let's revisit another classic, namely the Carpler-Strong algorithm,

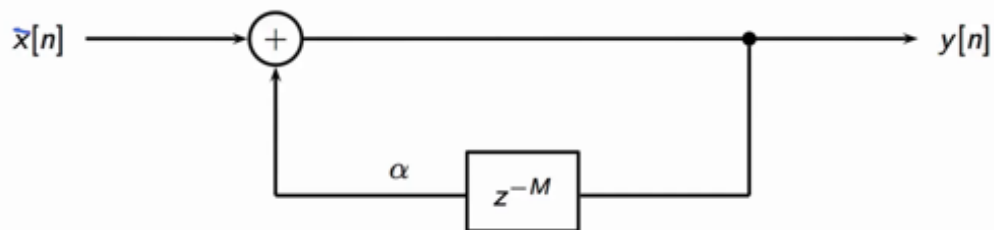
notes

summary

9m 33s



Karplus-Strong revisited, again!



$$y[n] = \alpha y[n - M] + x[n]$$

and let's try to analyze its behavior using the convolution theorem. Remember, the Karplus-Strong algorithm is initialized with a finite support signal x of support capital M , and then we use a feedback loop with a delay of capital M tabs to produce multiple copies of the original finite support signal scaled by an exponentially decaying factor α . So, for instance, if we initialize the algorithm with one period of the sawtooth wave,

notes

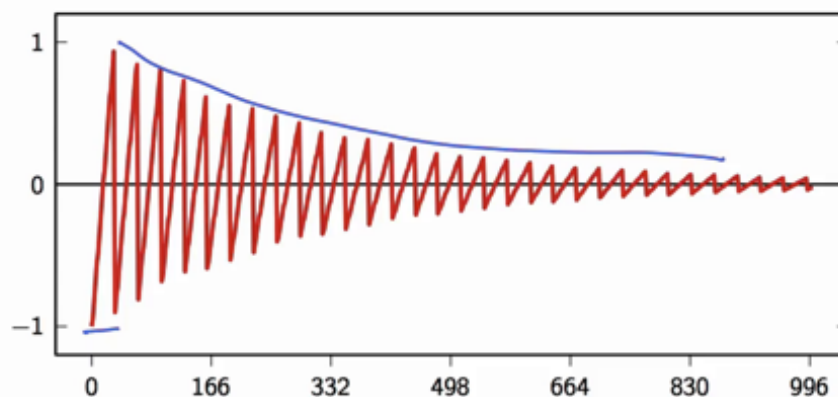
summary

10m 56s



KS revisited

$$y[n] = \alpha^{\lfloor n/M \rfloor} \bar{x}[n \bmod M] u[n]$$



what we get in the end is multiple repetitions of that period scaled by an exponentially decaying envelope. In the time domain, we can write this out explicitly as such.

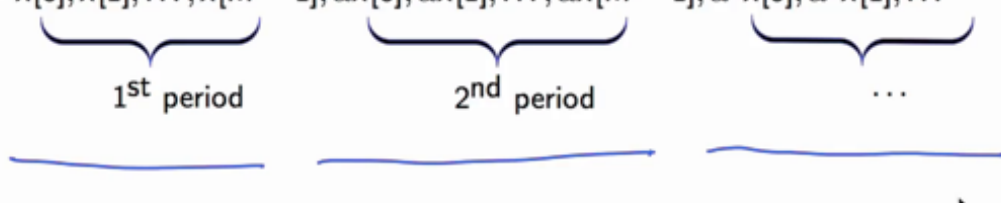
notes

summary

11m 28s



Karplus-Strong revisited

$$y[n] = \underbrace{\bar{x}[0], \bar{x}[1], \dots, \bar{x}[M-1]}_{\text{1st period}}, \underbrace{\alpha\bar{x}[0], \alpha\bar{x}[1], \dots, \alpha\bar{x}[M-1]}_{\text{2nd period}}, \underbrace{\alpha^2\bar{x}[0], \alpha^2\bar{x}[1], \dots}_{\dots}$$


Here we have the first period of the output signal is just a copy of the non-zero values of the finite support signal, followed by another copy of the non-zero values of the finite support signal scaled by alpha, followed by another copy scaled by alpha square, and so on and so forth. The key observation to analyze in the algorithm within the paradigm of the convolution theorem

notes

summary

11m 43s



DTFT of KS signal, using the convolution theorem

key observation:

$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$$

is to see that the output can be expressed as the convolution of the finite support input

notes

summary

12m 9s



DTFT of KS signal, using the convolution theorem

key observation:

$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$

with a sequence W of n that we build in the following way.

notes

summary

12m 18s



DTFT of KS signal, using the convolution theorem

key observation:

$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$$

$w[n]$ is equal to α^k for values of the index n that are the k th multiple of M , and it's zero otherwise. So if I was to draw the sequence, it would look like so.

notes

summary

12m 25s



DTFT of KS signal, using the convolution theorem

key observation:

$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$$

So it would be one in zero, it would be alpha in M, alpha square in 2M, and it would be zero in between.

notes

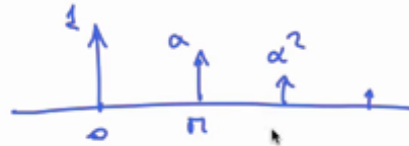
summary

12m 37s



DTFT of KS signal, using the convolution theorem

key observation:



$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$$

So a series of exponentially decaying deltas spaced M points apart.

notes

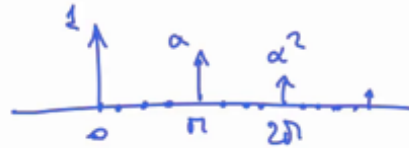
summary

12m 41s



DTFT of KS signal, using the convolution theorem

key observation:



$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$$

With this, we can write the convolution theorem and we say, okay, the output is simply in frequency

notes

summary

12m 52s



DTFT of KS signal, using the convolution theorem

key observation:

$$y[n] = \bar{x}[n] * w[n], \quad w[n] = \begin{cases} \alpha^k & \text{for } n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \bar{X}(e^{j\omega})W(e^{j\omega})$$

the product of the Fourier transform of the finite support signal times the Fourier transform of the signal W of n .

notes

summary

13m 4s



DTFT of KS signal, using the convolution theorem

$$\bar{X}(e^{j\omega}) = e^{-j\omega} \left(\frac{M+1}{M-1} \right) \frac{1 - e^{-j(M-1)\omega}}{(1 - e^{-j\omega})^2} - \frac{1 - e^{-j(M+1)\omega}}{(1 - e^{-j\omega})^2}$$
$$W(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega M}}$$

Well, and now we can proceed exactly like we did a few lectures back. If we consider the example of a sawtooth wave, we know that the Fourier transform of one period is this one,

notes

summary

13m 17s



DTFT of KS signal, using the convolution theorem

$$\bar{X}(e^{j\omega}) = e^{-j\omega} \left(\frac{M+1}{M-1} \right) \frac{1 - e^{-j(M-1)\omega}}{(1 - e^{-j\omega})^2} - \frac{1 - e^{-j(M+1)\omega}}{(1 - e^{-j\omega})^2}$$
$$W(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega M}}$$

whereas the Fourier transform of the sequence W of n is the rescaled Fourier transform of the exponentially decaying sequence, which means we will have several peaks happening in the minus π , π interval.

notes

summary

13m 30s



DTFT of KS signal, using the convolution theorem

$$\bar{X}(e^{j\omega}) = e^{-j\omega} \left(\frac{M+1}{M-1} \right) \frac{1 - e^{-j(M-1)\omega}}{(1 - e^{-j\omega})^2} - \frac{1 - e^{-j(M+1)\omega}}{(1 - e^{-j\omega})^2}$$

$$W(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega M}}$$



We put them together graphically, we have the Fourier transform in magnitude of the sawtooth period,

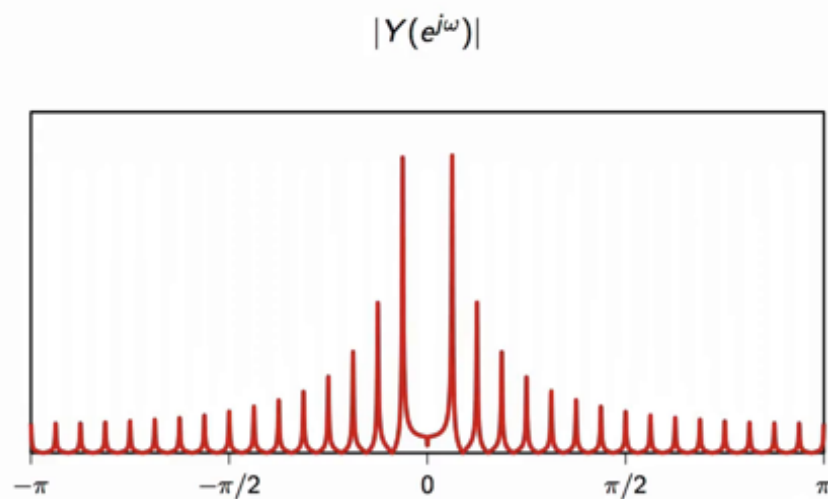
notes

summary

13m 45s



DTFT of KS



we have the Fourier transform in magnitude of the staggered exponential sequence, we take the product of the two and we obtain the same spectrum that we derived when we talked about the DTFT of the Karplus-Strong signal.

notes

summary

13m 52s

