



Course material

Course:

Digital Signal Processing II

Video:

2.1.5.c Ideal filters derived from the ideal lowpass filter

Concepts (extracted from automatically generated subtitles):

Omega c. Ideal low pass. Minus omega c. Time domain. Impulse response. Frequency response of the high pass. Bandpass filter. Ideal high pass filter. Whole thing. Sinc function. Delta function. Low pass filter. Cutoff frequency omega c. Frequency domain. Frequency response.



[to video sequence search](#)
(within Digital Signal Processing II.)

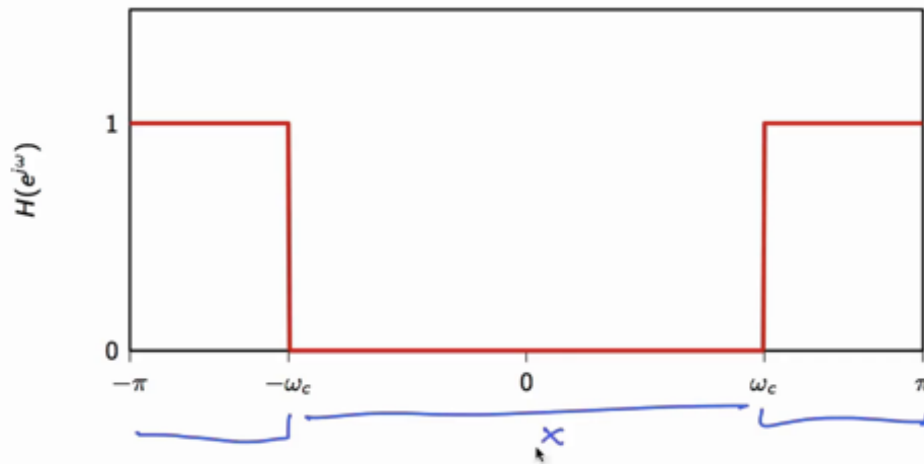


[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

Ideal highpass filter



Now that we have the ideal low pass in place, we can obtain a series of derived filters, such as the ideal high pass filter. You can see here from the plot that we are now preserving the frequencies from ω_c to π , and of course symmetrically those from $-\omega_c$ to $-\pi$, and we are killing everything in between. Now, formally, we can write the frequency response of the high pass as 1 for $|\omega| > \omega_c$

notes

summary

0m 0s



Ideal highpass filter

$$H_{hp}(e^{j\omega}) = \begin{cases} 1 & \text{for } \pi \geq |\omega| \geq \omega_c \\ 0 & \text{otherwise} \end{cases} \quad (2\pi\text{-periodicity implicit})$$

$$H_{hp}(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$$

$$h_{hp}[n] = \delta[n] - \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi} n\right)$$

between π and ω_c , and symmetrically from $-\pi$ to $-\omega_c$, and zero otherwise. The 2π periodicity is always implicit.

notes

summary

0m 22s



Ideal highpass filter

$$H_{hp}(e^{j\omega}) = \begin{cases} 1 & \text{for } \pi \geq |\omega| \geq \omega_c \\ 0 & \text{otherwise} \end{cases} \quad (2\pi\text{-periodicity implicit})$$

$$H_{hp}(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$$

$$h_{hp}[n] = \delta[n] - \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi} n\right)$$

We can also write that as 1 minus the complementary low pass. So if we have a low pass with cutoff frequency ω_c , and we take 1 minus that low pass, we obtain the high pass.

notes

summary

0m 37s



Ideal highpass filter

$$H_{hp}(e^{j\omega}) = \begin{cases} 1 & \text{for } \pi \geq |\omega| \geq \omega_c \\ 0 & \text{otherwise} \end{cases} \quad (2\pi\text{-periodicity implicit})$$

$$H_{hp}(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$$

$$h_{hp}[n] = \delta[n] - \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi} n\right)$$

⤴

So that the impulse response in time domain is actually a very simple modification of the impulse response of the complementary low pass. It's just the delta function minus the sinc function scaled by omega c over pi.

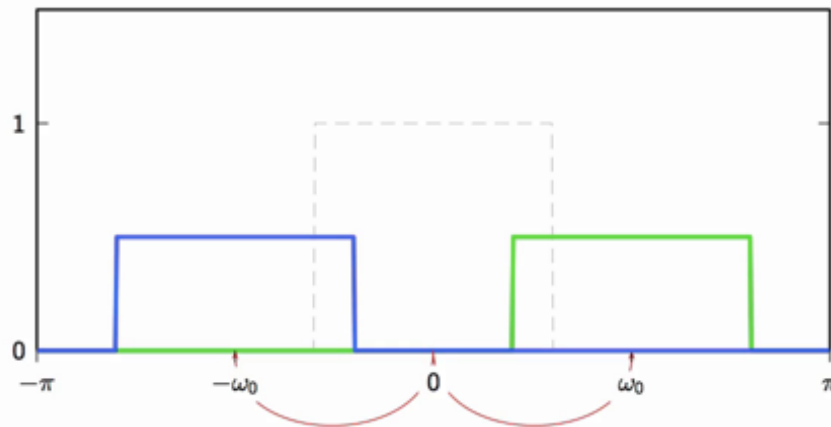
notes

summary

0m 49s



Ideal bandpass filter



Another variation is the bandpass filter. The bandpass filter preserves the frequencies in a band center around ω_0 and $2\omega_c$ wide, and the response is of course symmetric in the negative part of the spectrum. And we can obtain the bandpass filter starting from a prototype low pass filter with bandwidth again $2\omega_c$, and by modulating this low pass filter using a cosine modulation. So if we take a cosine at frequency ω_0 and we multiply that by the impulse response of the prototype low pass, we get one copy centered in ω_0 , $2\omega_c$ wide, and we get another copy in minus ω_0 . We sum the two, we scale them back to unit amplitude, and we have our ideal bandpass filter.

notes

summary

1m 1s



Ideal bandpass filter

$$H_{bp}(e^{j\omega}) = \begin{cases} 1 & \text{for } |\omega \pm \omega_0| \leq \omega_c \\ 0 & \text{otherwise} \end{cases} \quad \underbrace{(2\pi\text{-periodicity implicit})}$$

$$h_{bp}[n] = 2 \cos(\omega_0 n) \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi} n\right)$$

Formally, we can write the frequency response as 1 for ω plus or minus ω_0 less than ω_c in magnitude, and zero otherwise. Again, the whole thing is 2π periodic, and if we look at the time domain,

notes

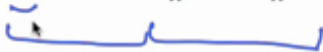
summary

1m 57s



Ideal bandpass filter

$$H_{bp}(e^{j\omega}) = \begin{cases} 1 & \text{for } |\omega \pm \omega_0| \leq \omega_c \\ 0 & \text{otherwise} \end{cases} \quad (2\pi\text{-periodicity implicit})$$

$$h_{bp}[n] = 2 \cos(\omega_0 n) \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi} n\right)$$


we see that we have the standard impulse response of the prototype low pass filter with cutoff frequency ω_c modulated by the cosine of $\omega_0 n$, and the factor 2 here scales the filter back to unit amplitude in the frequency domain.

notes

summary

2m 13s

