

Course material

Course:

## Digital Signal Processing II

Video:

### 2.2.1.a Impulse truncation (and the Gibbs phenomenon)

Concepts (extracted from automatically generated subtitles):

**Impulse response. Cutoff frequency  $\omega_c$ . Ideal filter. Mean square error. Ideal impulse response. Frequency domain. Capital  $n$ . Design of a low-pass filter.  $\pi$  times. Justification of this method. Low-pass. Finite support  $h$ . Norm of the error. Frequency response of the approximated filter. Good idea.**



[to video sequence search](#)  
(within Digital Signal Processing II.)



[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

## How can we approximate an ideal lowpass?

Idea #1:

- ▶ pick  $\omega_c$
- ▶ compute ideal impulse response  $h[n]$
- ▶ truncate  $h[n]$  to a finite-support  $\hat{h}[n]$
- ▶  $\hat{h}[n]$  defines an FIR filter

So you remember an ideal filter is not realizable in practice because the impulse response is a two-sided infinite support sequence. So we're going to try to approximate it by starting from the ideal impulse response and by manipulating this response into something that we can actually deal with. In the next examples we will concentrate on the design of a low-pass filter but clearly the same techniques can be applied to any type of ideal filter. So the first idea is the following. We pick a cutoff frequency  $\omega_c$ , we compute the ideal impulse response for the

notes

summary

0m 0s



## How can we approximate an ideal lowpass?

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- ▶  $\hat{h}[n]$  defines an FIR filter

low-pass analytically, we truncate  $h$  of  $n$  to a finite support  $\hat{h}$  of  $n$ . This  $\hat{h}$  of  $n$  defines an FIR filter that we can use. For the time being we don't worry

notes

summary

0m 33s



## Approximation by truncation

FIR approximation of length  $M = 2N + 1$ :

$$\hat{h}[n] = \begin{cases} \frac{\omega_c}{\pi} \operatorname{sinc}\left(\frac{\omega_c}{\pi} n\right) & |n| \leq N \\ 0 & \text{otherwise} \end{cases}$$

about causality so we preserve the symmetry around zero of the impulse

notes

summary

0m 45s



## Approximation by truncation

FIR approximation of length  $M = 2N + 1$ :

$$\hat{h}[n] = \begin{cases} \frac{\omega_c}{\pi} \operatorname{sinc}\left(\frac{\omega_c}{\pi} n\right) & |n| \leq N \\ 0 & \text{otherwise} \end{cases}$$

response and we approximate the ideal filter with an FIR of length  $2N + 1$  centered around zero. And so here  $\hat{h}[n]$  is equal to  $\frac{\omega_c}{\pi}$  times the sinc of  $\frac{\omega_c}{\pi} n$  for  $|n| \leq N$  and zero everywhere else. A justification of this method is the fact

notes

summary

0m 49s



## Approximation by truncation

FIR approximation of length  $M = 2N + 1$ :

$$\hat{h}[n] = \begin{cases} \frac{\omega_c}{\pi} \operatorname{sinc}\left(\frac{\omega_c}{\pi} n\right) & |n| \leq N \\ 0 & \text{otherwise} \end{cases}$$

that if we compute the mean square error

notes

summary

1m 12s



## Why it could be a good idea

$$\begin{aligned}\text{MSE} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega}) - \hat{H}(e^{j\omega})|^2 d\omega \\ &= \|H(e^{j\omega}) - \hat{H}(e^{j\omega})\|^2 \\ &= \|h[n] - \hat{h}[n]\|^2 \\ &= \sum_{n=-\infty}^{\infty} |h[n] - \hat{h}[n]|^2\end{aligned}$$

MSE is minimized by symmetric impulse truncation around zero

between the original filter and the approximation we have that the norm of the error in the frequency domain is the integral from minus pi to pi of the

notes

summary

1m 13s



## Why it could be a good idea

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MSE is minimized by symmetric impulse truncation around zero

frequency response of the ideal filter minus the frequency response of the approximated filter squared. This is equal to the norm squared of the

notes

summary

1m 25s



## Why it could be a good idea

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MSE is minimized by symmetric impulse truncation around zero

difference between the two frequency responses and thanks to Parseval's theorem this is also the norm squared of

notes

summary

1m 33s



## Why it could be a good idea

$$\begin{aligned}\text{MSE} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega}) - \hat{H}(e^{j\omega})|^2 d\omega \\ &= \|H(e^{j\omega}) - \hat{H}(e^{j\omega})\|^2 \\ &= \|h[n] - \hat{h}[n]\|^2 \\ &= \sum_{n=-\infty}^{\infty} |h[n] - \hat{h}[n]|^2\end{aligned}$$

MSE is minimized by symmetric impulse truncation around zero

the difference between the impulse responses in the time domain. And so we can write that the mean square error is simply the sum from n that goes to minus

notes

summary

1m 40s



## Why it could be a good idea

$$\begin{aligned}\text{MSE} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega}) - \hat{H}(e^{j\omega})|^2 d\omega \\ &= \|H(e^{j\omega}) - \hat{H}(e^{j\omega})\|^2 \\ &= \|h[n] - \hat{h}[n]\|^2 \\ &= \sum_{n=-\infty}^{\infty} |h[n] - \hat{h}[n]|^2\end{aligned}$$



MSE is minimized by symmetric impulse truncation around zero

infinity to plus infinity of the difference between the two impulse responses squared. The impulse response of the ideal low pass is the sinc and you remember it decays monotonically as 1 over n. So the mean

notes

summary

1m 49s



## Why it could be a good idea

$$\begin{aligned}\text{MSE} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega}) - \hat{H}(e^{j\omega})|^2 d\omega \\ &= \|H(e^{j\omega}) - \hat{H}(e^{j\omega})\|^2 \\ &= \|h[n] - \hat{h}[n]\|^2 \\ &= \sum_{n=-\infty}^{\infty} |h[n] - \hat{h}[n]|^2\end{aligned}$$



MSE is minimized by symmetric impulse truncation around zero

square error is clearly minimized if we pick values for  $\hat{h}$  of  $n$  from an interval that is symmetric around zero because in this way we're actually

notes

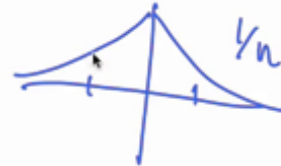
summary

2m 1s



## Why it could be a good idea

$$\begin{aligned}\text{MSE} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega}) - \hat{H}(e^{j\omega})|^2 d\omega \\ &= \|H(e^{j\omega}) - \hat{H}(e^{j\omega})\|^2 \\ &= \|h[n] - \hat{h}[n]\|^2 \\ &= \sum_{n=-\infty}^{\infty} |h[n] - \hat{h}[n]|^2\end{aligned}$$



MSE is minimized by symmetric impulse truncation around zero

killing in that summation the largest

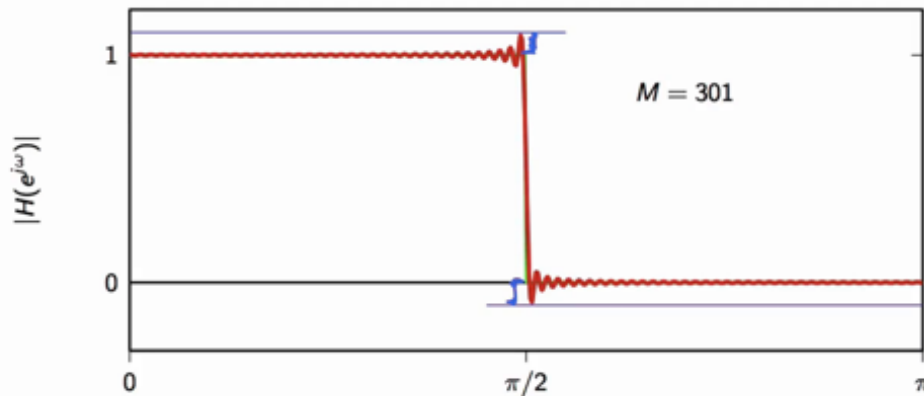
notes

summary

2m 12s



## Why it's not such a good idea



terms. So we have seen why impulse truncation might be a good idea but it's time to look at why it might not be such a good idea after all. And to appreciate that we have to look at the frequency response of the approximate filter. So here in this plot we show the frequency response of an ideal low pass filter where we choose  $\omega_c$  equal to  $\pi/2$  and superimposed we're going to show the frequency response of the approximate filter for increasing values of capital M. So we start with M equal to 9 which is not really a very long filter and we see that the approximation is not particularly good. So we increase the number of taps of the filter and as we add more and more points to the impulse response we see that the characteristic in red approaches the characteristic in green. But it looks as though the maximum error near the transition point for the frequency response never really goes down in spite of the number of points that we use for the approximation. We can try and push the number of points even higher and we see that even for 300 points the maximum error around  $\omega_c$  is always of the same amplitude. We can prove mathematically that the maximum

### notes

### summary

2m 13s



## The Gibbs phenomenon

The maximum error around the cutoff frequency  
is around 9% of the height of the jump  
*regardless of  $N$*

error around the cutoff frequency is about 9% of the height of the jump regardless of the number of points. This is a standard result in approximation theory known under the name of Gibbs phenomenon. It always happens when we try to approximate this continuous function.

### notes

### summary

3m 30s

