



Course material

Course:

Digital Signal Processing II

Video:

2.2.3 Intuitive IIR designs

Concepts (extracted from automatically generated subtitles):

Frequency domain. Simple filters. Pole zero plot. Dc removal filter. Low order sections. Signal processing problems. Classic use case. Low frequencies. High frequencies. Frequency response. Transfer function. Simple structures. Zero mean. Hum removal filter. Fourier transform.



[to video sequence search](#)
(within Digital Signal Processing II.)



[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

Simple, useful filters

- ▶ many signal processing problems can be solved using simple filters
- ▶ we have seen simple lowpass filters already (Moving Average, Leaky Integrator)
- ▶ simple (low order) transfer functions allow for intuitive design and tuning

Many signal processing problems can be solved using very simple filters, so it is important to master the simple structures. We have already seen some examples such as the Leaky Integrator and the Moving Average. In general, when we use low order sections, we can intuitively understand what goes on in the frequency domain just by looking at the pole zero plot. Consider the problem of designing a simple low pass.

notes

summary

0m 0s



Simple lowpass

- ▶ let only low frequencies pass
- ▶ used to remove high frequency components (e.g. noise)
- ▶ useful in audio, communication, control systems
- ▶ we know a simple answer: leaky integrator

We want the low frequencies to go through, we want to remove the high frequencies, and we can use such a filter in audio problems in communication engineering or in control systems. We have seen a very simple example of a low pass, which is the Leaky Integrator.

notes

summary

0m 19s



Simple lowpass

- ▶ let only low frequencies pass
- ▶ used to remove high frequency components (e.g. noise)
- ▶ useful in audio, communication, control systems
- ▶ we know a simple answer: leaky integrator

But now we know the transfer function by heart, this one, and we also know the CCDE.

notes

summary

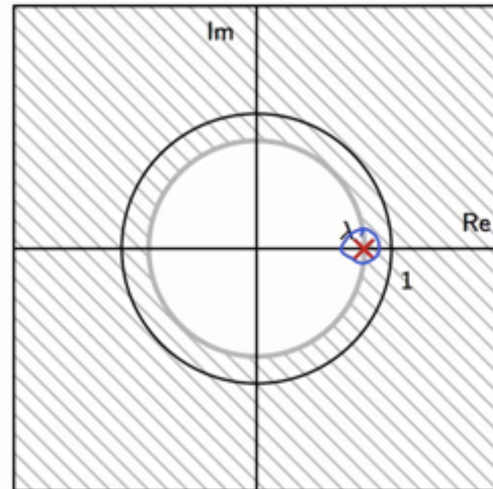
0m 34s



Leaky Integrator

$$H(z) = \frac{(1 - \lambda)}{1 - \lambda z^{-1}}$$

$$y[n] = (1 - \lambda)x[n] + \lambda y[n - 1]$$



We can plot the poles and the zeros of the Leaky Integrator on the complex plane, and we find that there's only one pole in lambda, so when lambda is less than one, the filter is stable. In the frequency domain, the frequency response looks like this,

notes

summary

0m 38s



Resonator

- ▶ a resonator is a narrow bandpass filter
- ▶ used to detect the presence of a sinusoid of a given frequency
- ▶ useful in communication systems and telephony (DTMF)
- ▶ idea: shift the passband of the Leaky Integrator!

and it gets more and more concentrated around the origin as λ gets closer to one. The block diagram for the filter is a simple feedback loop with a unit delay that pipes back the output into the input. Now that we know how to design a Leaky Integrator, we can adapt the design to obtain a resonator. A resonator is a narrow bandpass filter that is often used to detect the presence of a sinusoid

notes

summary

0m 55s



Resonator

- ▶ a resonator is a narrow bandpass filter
- ▶ used to detect the presence of a sinusoid of a given frequency
- ▶ useful in communication systems and telephony (DTMF)
- ▶ idea: shift the passband of the Leaky Integrator!

of a given frequency. A classic circuit that uses resonators is a detector for DTMF signals. DTMF stands for Dual Tone Multi-Frequency Signaling,

notes

summary

1m 21s



Resonator

dual-tone

M

- ▶ a resonator is a narrow bandpass filter
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- ▶ idea: shift the passband of the Leaky Integrator!

and it is the way an analog telephones communicates to the central office the dial number.

notes

summary

1m 33s



Resonator

dual-tone
multi-freq

- ▶ a resonator is a narrow bandpass filter
- ▶ used to detect the presence of a sinusoid of a given frequency
- ▶ useful in communication systems and telephony (DTMF)
- ▶ idea: shift the passband of the Leaky Integrator!



Whenever you press a number on the dial pad of a telephone, you generate two frequencies that will have to be decoded at the central office to determine which key you have pressed. The idea is to obtain a resonator by shifting the passband of the Leaky Integrator.

notes

summary

1m 37s



Resonator

- ▶ a resonator is a narrow bandpass filter
- ▶ used to detect the presence of a sinusoid of a given frequency
- ▶ useful in communication systems and telephony (DTMF)
- ▶ idea: shift the passband of the Leaky Integrator!

Remember the Leaky Integrator has a pole in λ here, and the idea is to move the pole

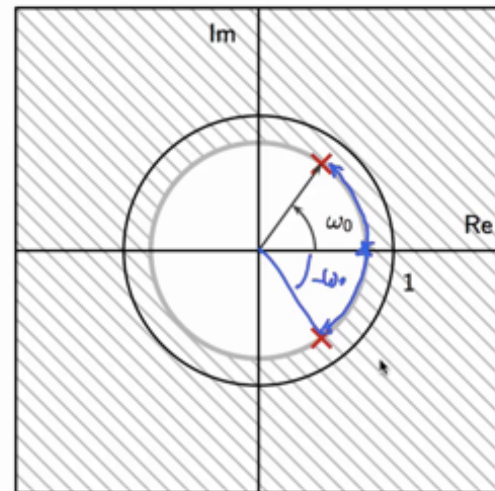
notes

summary

1m 59s



Resonator



radially around the circle of radius λ to shift the passband at the frequency that we're interested in selecting. Since we want a real filter, we also have to create a complex conjugate pole at an angle that is minus ω_0 . The transfer function of this system is simply

notes

summary

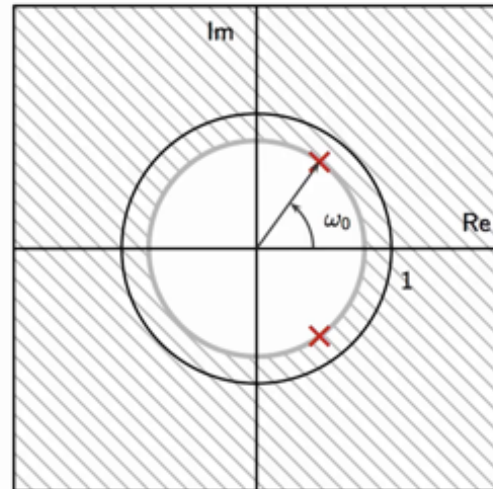
2m 4s



Resonator

$$H(z) = \frac{G_0}{(1 - pz^{-1})(1 - p^*z^{-1})}$$

$p = \lambda e^{j\omega_0}$



$H(z)$ is equal to some gain factor that we will determine later, divided by two first-order terms that correspond to the two poles in the complex plane. As for the poles, we can conveniently express them as $\lambda e^{j\omega_0}$, where ω_0 is the frequency we want to select. In the time domain, the output is the gain times the input minus $a_1 y[n-1]$ minus $a_2 y[n-2]$.

notes

summary

2m 29s

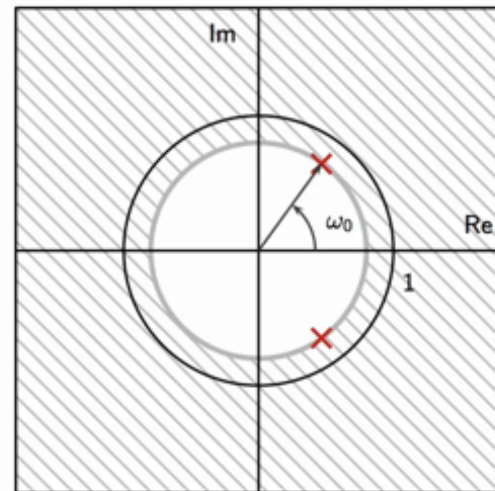


Resonator

$$H(z) = \frac{G_0}{(1 - pz^{-1})(1 - p^*z^{-1})}$$

$$p = \lambda e^{j\omega_0}$$

$$y[n] = \underbrace{G_0 x[n]}_{\text{input}} - \underbrace{a_1 y[n-1]}_{\text{feedback}} - \underbrace{a_2 y[n-2]}_{\text{feedback}}$$



n minus 2. So we have two output samples that are delayed and fed back into the input. The coefficients a_1 and a_2 we can determine if we work out the product of these first-order terms. Let's work

notes

summary

2m 53s

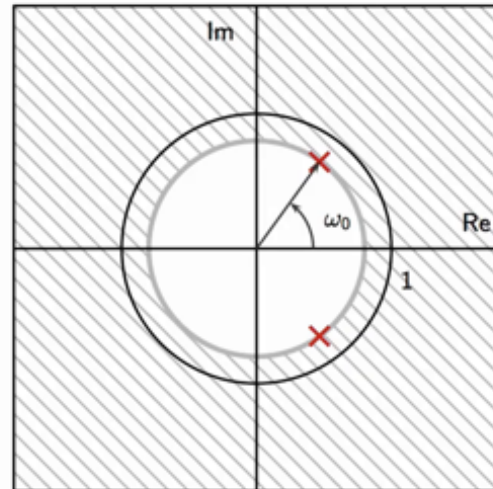


Resonator

$$H(z) = \frac{G_0}{(1 - pz^{-1})(1 - p^*z^{-1})}$$

$$p = \lambda e^{j\omega_0}$$

$$y[n] = G_0x[n] - a_1y[n-1] - a_2y[n-2]$$



a little bit on the transfer function. We start in factor form, and then we can work out the product

notes

summary

3m 12s



Resonator

$$H(z) = \frac{G_0}{(1 - pz^{-1})(1 - p^*z^{-1})}, \quad p = \lambda e^{j\omega_0}$$

$$= \frac{G_0}{1 - 2\Re\{p\}z^{-1} + |p|^2 z^{-2}}$$

$$= \frac{G_0}{1 - \underbrace{2\lambda \cos \omega_0}_{a_1} z^{-1} + \underbrace{|\lambda|^2}_{a_2} z^{-2}}$$

$$a_1 = 2\lambda \cos \omega_0$$

$$a_2 = -|\lambda|^2$$

to obtain this canonical form in which we can find the coefficients that we will have to use in the CCD. So a_1 is 2 times lambda cosine of omega zero, whereas a_2 is minus magnitude of lambda square. In

notes

summary

3m 16s



Resonator

$$\begin{aligned} H(z) &= \frac{G_0}{(1 - pz^{-1})(1 - p^*z^{-1})}, & p &= \lambda e^{j\omega_0} \\ &= \frac{G_0}{1 - 2\Re\{p\}z^{-1} + |p|^2 z^{-2}} \\ &= \frac{G_0}{1 - 2\lambda \cos \omega_0 z^{-1} + |\lambda|^2 z^{-2}} \end{aligned}$$

$$a_1 = 2\lambda \cos \omega_0$$

$$a_2 = -|\lambda|^2$$

the frequency domain, the resonator indeed behaves as we expected. We have now two peaks in the

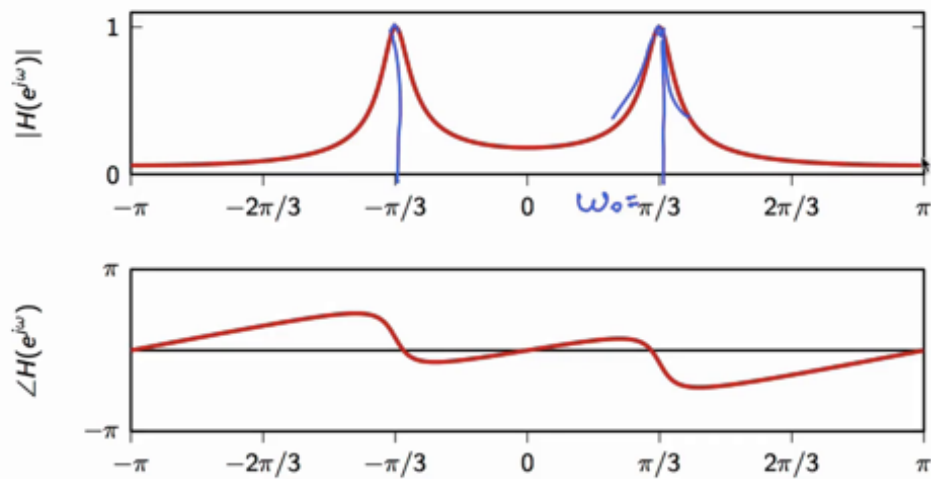
notes

summary

3m 31s



Resonator, $\lambda = 0.9, \omega_0 = \pi/3$



frequency response that are centered on the frequency of interest ω_0 , which in this example is equal to $\pi/3$. The phase is again non-linear as we expect from a Leake integrator, albeit modified. The problem with this resonator is that it's not very selective because the pass bands are quite spread out. We can improve the selectivity of the resonator by choosing a

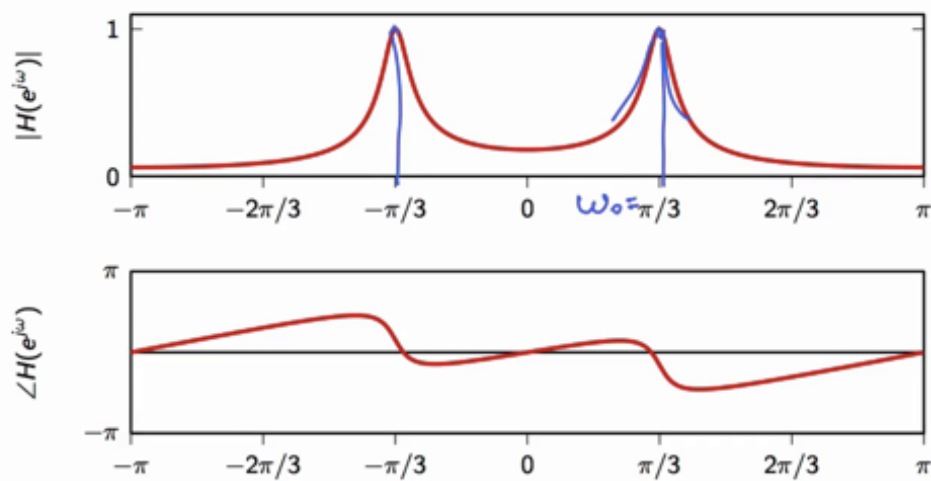
notes

summary

3m 37s



Resonator, $\lambda = 0.9, \omega_0 = \pi/3$



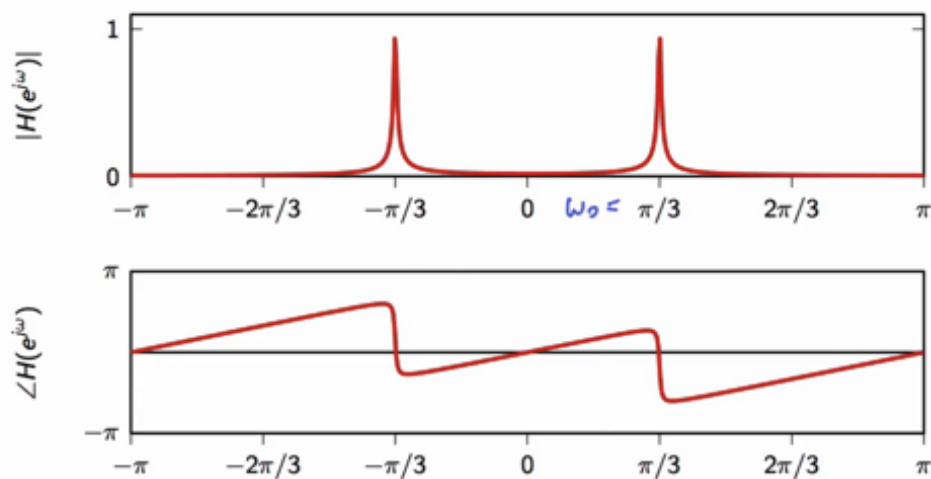
value for lambda that is closer to 1, and indeed if we choose lambda equal to 0.99 for the same

notes

summary

4m 11s





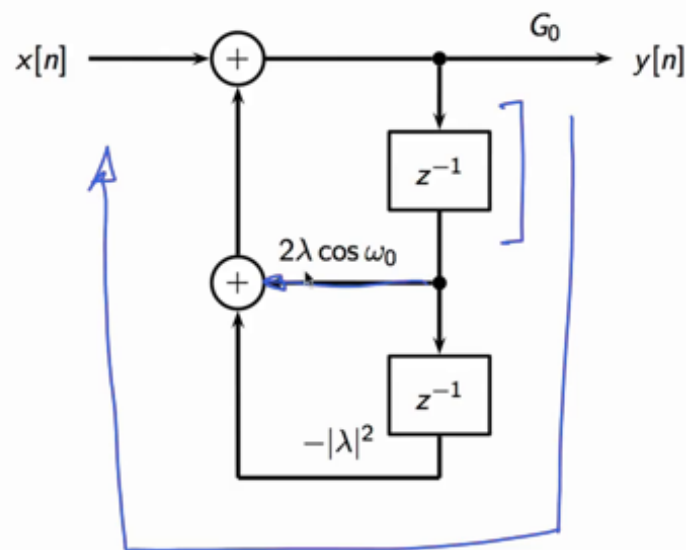
frequency of ω_0 equal to $\pi/3$, we obtain two peaks that are much more narrow and centered around the frequency of interest. As for the filter structure, this time we have

notes

summary

4m 17s





two delays. It's a simple feedback loop once again, but a second-order feedback loop in which the input is first delayed by the first delay and then piped back via the factor $2\lambda \cos \omega_0$ and then delayed again and piped back via the factor magnitude of λ square.

notes

summary

4m 33s



DC removal

- ▶ a DC-balanced signal has zero sum: $\lim_{N \rightarrow \infty} \sum_{n=-N}^N x[n] = 0$
i.e. there is no Direct Current component
- ▶ its DTFT value at zero is zero
- ▶ we want to remove the DC bias from a non zero-centered signal
- ▶ we want to kill the frequency component at $\omega = 0$

Let's look now at a DC removal filter. DC stands for direct current and when we say that the signal

notes

summary

4m 49s



DC removal

- ▶ a DC-balanced signal has zero sum: $\lim_{N \rightarrow \infty} \sum_{n=-N}^N x[n] = 0$

i.e. there is no Direct Current component

- ▶ its DTFT value at zero is zero

X

- ▶ we want to remove the DC bias from a non zero-centered signal

- ▶ we want to kill the frequency component at $\omega = 0$

is DC balanced we mean that the signal has zero sum or zero mean if you will. The reason why we want signals to be DC balanced is because a DC offset carries no information but throws the circuits off by wasting current in the loop. A DC balanced signal will have a Fourier transform which is equal to zero for omega equal to zero. Remember if you think of the definition of the

notes

summary

5m 1s



DC removal

- ▶ a DC-balanced signal has zero sum: $\lim_{N \rightarrow \infty} \sum_{n=-N}^N x[n] = 0$

i.e. there is no Direct Current component

- ▶ its DTFT value at zero is zero

$$X(e^{j\omega}) = \sum x[n] e^{j\omega n}$$

- ▶ we want to remove the DC bias from a non zero-centered signal
- ▶ we want to kill the frequency component at $\omega = 0$

Fourier transform you have that and for omega equal to zero this is simply the sum of the signal

notes

summary

5m 30s



DC removal

- ▶ a DC-balanced signal has zero sum: $\lim_{N \rightarrow \infty} \sum_{n=-N}^N x[n] = 0$
i.e. there is no Direct Current component
- ▶ its DTFT value at zero is zero
- ▶ we want to remove the DC bias from a non zero-centered signal
- ▶ we want to kill the frequency component at $\omega = 0$

samples. So in order to remove the DC bias from a non-zero centered signal we need to kill the frequency component at omega equal to zero. The way to do that is rather straightforward.

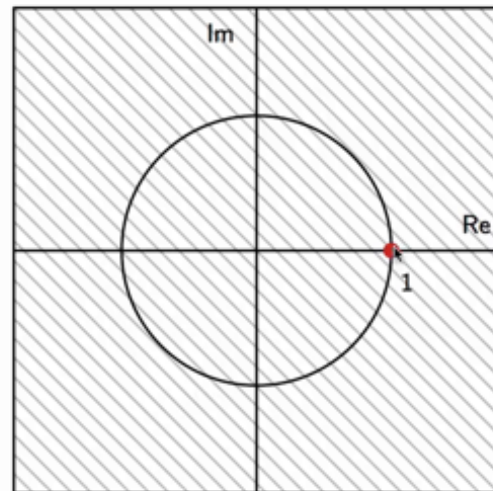
notes

summary

5m 40s



DC removal



We simply place a zero at z equal to one which is the point in the complex plane that corresponds to a frequency of ω equal to zero. Transfer function is very easy. It's one minus z to the

notes

summary

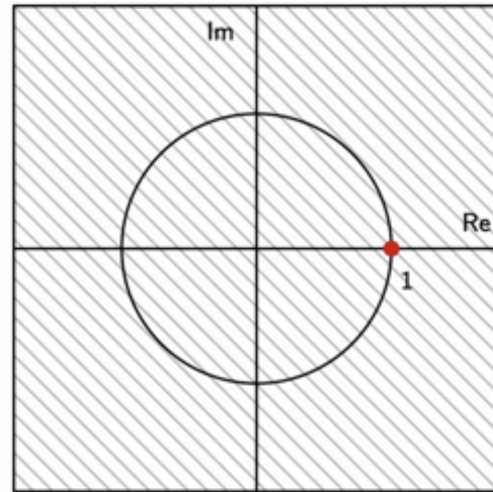
5m 54s



DC removal

$$H(z) = 1 - z^{-1}$$

$$y[n] = x[n] - x[n-1]$$



minus one and the CCDE is equally easy. The output is computed as the difference of two subsequent samples so $x[n] - x[n-1]$. If we plot the frequency response

notes

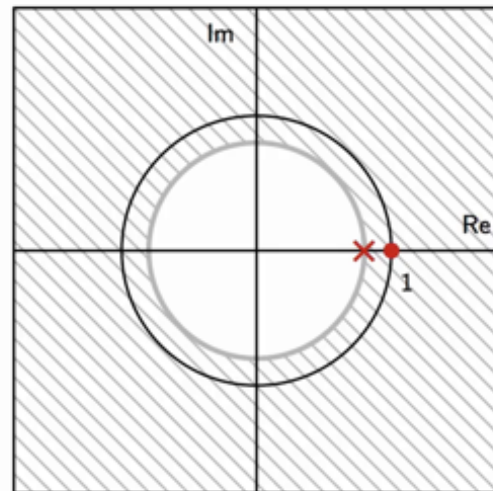
summary

6m 1s



DC removal, improved

$$H(z) = \frac{1 - z^{-1}}{1 - \lambda z^{-1}}$$



this is the characteristic we obtain. Indeed the Fourier transform will be set to zero in zero. However this is definitely not an acceptable characteristic because it introduces a very big attenuation over almost the entirety of the frequency support. What we want really is a notch which is to say we want to put the Fourier transform at zero and zero and leave the rest of the spectrum untouched. So we need to somehow improve on the design and if we remember the circus tent method we know that we can push up the z transform by putting a pole in the vicinity of the zero. So we try and do that and we combine therefore the effect of a zero in one with the effect of a pole close to one and inside the unit circle for obvious reasons of stability. So this is really the combination of a notch in zero which is given by the numerator here and of a leaky integrator at the denominator. In the time domain we have that the output is now λ times the output

notes

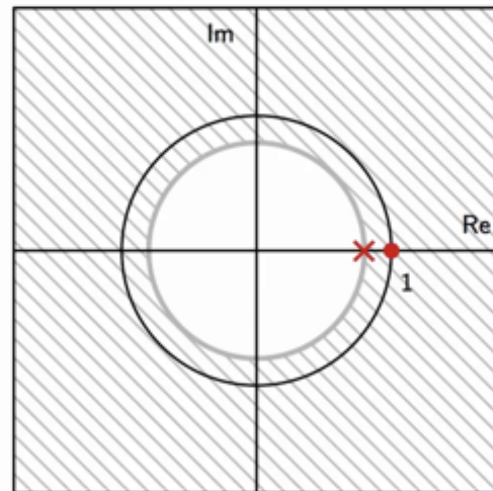
summary

6m 18s



DC removal, improved

$$H(z) = \frac{1 - z^{-1}}{1 - \lambda z^{-1}}$$
$$y[n] = \lambda y[n-1] + x[n] - x[n-1]$$



at n minus one plus x of n minus x of n minus one. If we plot the modified notch we obtain a characteristic

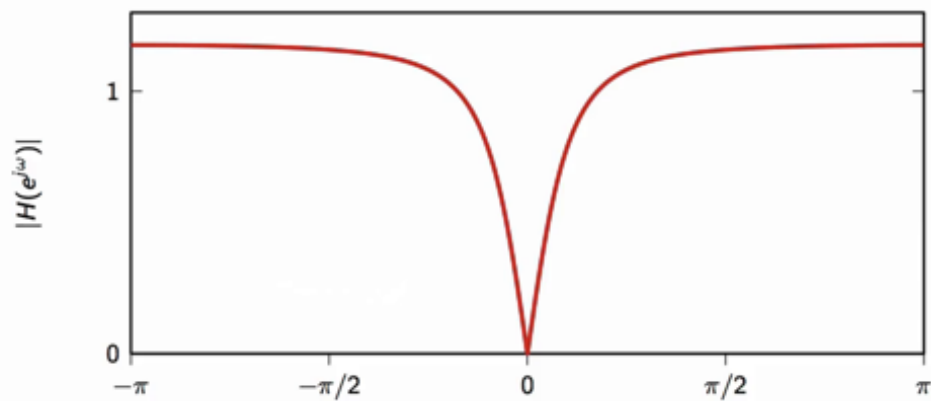
notes

summary

7m 19s



DC notch, $\lambda = 0.7$



that varies with different values of lambda and as lambda grows closer to one the notch becomes

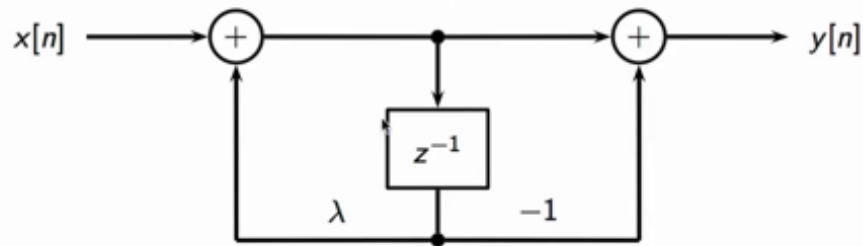
notes

summary

7m 25s



DC notch, filter structure



more narrow. And so here for instance the example is for lambda equal to 0.98 and we have achieved a very selective DC removal. The filter structure is very simple we will exploit the direct form two that we have seen in the previous module and share the delay between the feed forward and the

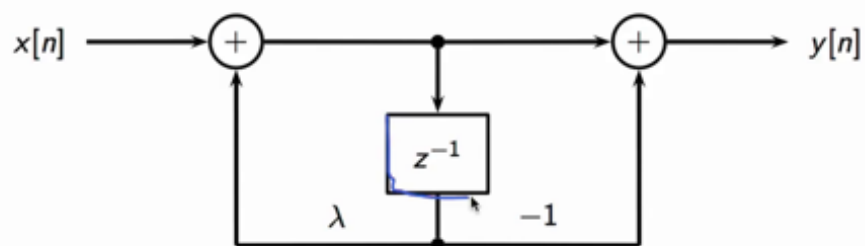
notes

summary

7m 37s



DC notch, filter structure



feedback part of the filter. Finally let's consider the hum removal filter. The hum removal is to the

notes

summary

8m 0s



Hum removal

- ▶ similar to DC removal but we want to remove a specific nonzero frequency
- ▶ very useful for musicians: amplifiers for electric guitars pick up the hum from the electric mains (50Hz in Europe and 60Hz in North America)
- ▶ we need to tune the hum removal according to country

DC notch what the resonator was to the leaky integrator. In other words the hum removal is a DC notch whose stop band we shift in order to be able to eliminate a specific frequency. A classic use case for the hum removal is a music. Amplifiers and electric guitars tend to pick up

notes

summary

8m 1s



Hum removal

- ▶ similar to DC removal but we want to remove a specific nonzero frequency
- ▶ very useful for musicians: amplifiers for electric guitars pick up the hum from the electric mains (50Hz in Europe and 60Hz in North America)
- ▶ we need to tune the hum removal according to country

the hum generated by the AC current in the mains. That frequency is 50 hertz in Europe and 60 hertz in North America. 50 or 60 hertz are well within the hidden range so not only is the hum audible but it is also particularly irritating. So we need a filter to remove the hum and we need to make

notes

summary

8m 23s



Hum removal

- ▶ similar to DC removal but we want to remove a specific nonzero frequency
- ▶ very useful for musicians: amplifiers for electric guitars pick up the hum from the electric mains (50Hz in Europe and 60Hz in North America)
- ▶ we need to tune the hum removal according to country

sure that we can tune this filter according to whether we're playing in Europe or in North America. The strategy is going to be exactly the same as the one we used to derive the resonator. We take

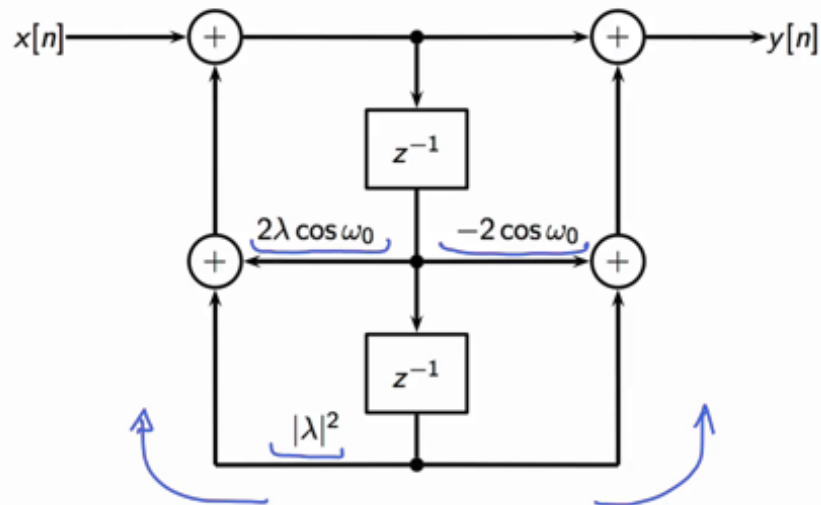
notes

summary

8m 49s



Hum removal, filter structure



a DC notch that has a zero in one and a pole next to it and we move both the pole and the zero along a circle of radius λ and one respectively. Of course in order to have a real system we have to introduce a complex conjugate pole and a complex conjugate zero at an angle that is minus ω_0 . When we do that we obtain a second order transfer function with a second order numerator and a second order denominator. If we plot the frequency response we get the characteristic that we're looking for and we're placing the notches now at a frequency of interest ω_0 . The structure is a standard direct form two implementation where we have shared the delays between the feed forward and the feedback part of the filter and the coefficients can be worked out by carrying out the products between the first order terms in the transfer function.

notes

summary

8m 58s

