



Course material

Course:

Digital Signal Processing II

Video:

2.2.4.b IIR design

Concepts (extracted from automatically generated subtitles):

Filter design. IIR design techniques. Filter order. Desired cutoff frequency. Low pass prototype. Digital signal processing. Lots of nice designs. Band pass. Type of filter. Magnitude response. Parameters. Provided filter. Ready-made routines. Analog designs. Cutoff frequency.



[to video sequence search](#)
(within Digital Signal Processing II.)



[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

IIR: conversion of analog design

Filter design was an established art long before digital processing appeared

- ▶ lots of nice analog filters exist
- ▶ methods exist to "translate" the analog design into a rational transfer function
- ▶ most numerical packages (Matlab, etc.) provide ready-made routines
- ▶ design involves specifying some parameters and testing that the specs are fulfilled

Let's begin our survey by looking at some IAR design techniques. So filter design was an established art long before digital signal processing came around and lots of nice designs were developed along the years using discrete electronics.

notes

summary

0m 0s



IIR: conversion of analog design

Filter design was an established art long before digital processing appeared

- ▶ lots of nice analog filters exist
- ▶ methods exist to “translate” the analog design into a rational transfer function
- ▶ most numerical packages (Matlab, etc.) provide ready-made routines
- ▶ design involves specifying some parameters and testing that the specs are fulfilled

There are some methods that can translate analog designs into rational transfer function. We will not look at the details of these methods, but most numerical packages such as MATLAB

notes

summary

0m 13s



IIR: conversion of analog design

Filter design was an established art long before digital processing appeared

- ▶ lots of nice analog filters exist
- ▶ methods exist to “translate” the analog design into a rational transfer function
- ▶ most numerical packages (Matlab, etc.) provide ready-made routines
- ▶ design involves specifying some parameters and testing that the specs are fulfilled

provide you with ready-made routines that will help you design filters according to this classic template. The design often involves a trial and error phase. You specify some parameters from the beginning, such as the type of filter and the desired cutoff frequency, and then you guess other parameters, usually the filter order, you run the routine and you verify that the provided filter fulfills the specs.

notes

summary

0m 25s



Butterworth lowpass

Magnitude response:

- ▶ maximally flat
- ▶ monotonic over $[0, \pi]$

Design parameters:

- ▶ order N
- ▶ cutoff frequency

Test values:

- ▶ width of transition band
- ▶ passband error

If that is not the case, you will probably have to change and increase the filter order until the specs are met. Let's now look at a few classic filters mutated from the analog world. We will concentrate on a low pass prototype, but be aware that design techniques exist for high pass and band pass as well. The first filter we will look at is the Butterworth low pass. The Butterworth filter has a magnitude response which is maximally flat and it is monotonic over the zero pi interval.

notes

summary

0m 49s



Butterworth lowpass

Magnitude response:

- ▶ maximally flat
- ▶ monotonic over $[0, \pi]$

Design parameters:

- ▶ order N ✓
- ▶ cutoff frequency

Test values:

- ▶ width of transition band
- ▶ passband error

The design parameters are very simple, the order of the filter and the desired cutoff frequency.

notes

summary

1m 13s



Butterworth lowpass

Magnitude response:

- ▶ maximally flat
- ▶ monotonic over $[0, \pi]$

Design parameters:

- ▶ order N ✓
- ▶ cutoff frequency ✓

Test values:

- ▶ width of transition band
- ▶ passband error

You run the algorithm and you get a prototype filter. You test the filter with respect to the width of the transition band and the pass band error.

notes

summary

1m 25s



Butterworth lowpass

Magnitude response:

- ▶ maximally flat
- ▶ monotonic over $[0, \pi]$

Design parameters:

- ▶ order N
- ▶ cutoff frequency

Test values:

- ▶ width of transition band ✓
- ▶ passband error ✓

If any of these parameters do not fulfill the specifications, then you increase the order and you run the algorithm again. The frequency response of the Butterworth filter looks like this.

notes

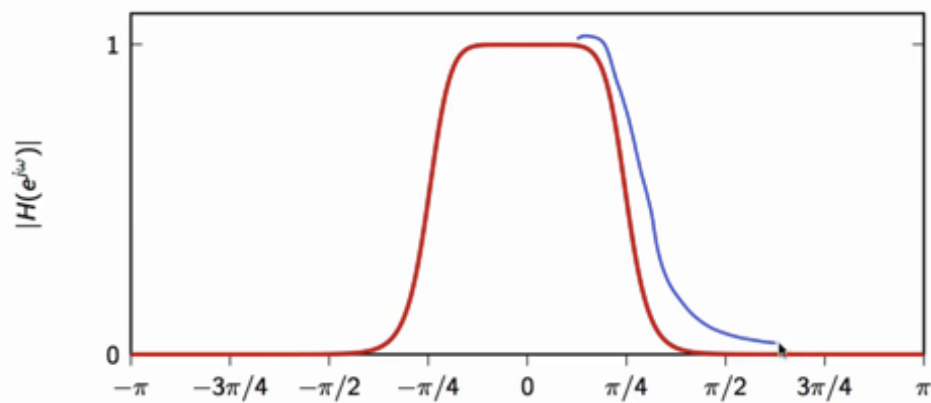
summary

1m 30s



Butterworth lowpass example

$$N = 4, \omega_c = \pi/4$$



It is, as we said, a monotonic curve that decays smoothly from zero to pi.

notes

summary

1m 41s



Chebyshev lowpass

Magnitude response:

- ▶ equiripple in passband
- ▶ monotonic in stopband

Design parameters:

- ▶ order N
- ▶ passband max error
- ▶ cutoff frequency

Test values:

- ▶ width of transition band
- ▶ stopband error

This is an example of order four with a cutoff frequency of $\pi/4$. The Chebyshev low pass has a magnitude response which is equiripple in the pass band and monotonic in the stop band.

notes

summary

1m 49s



Chebyshev lowpass

Magnitude response:

- ▶ equiripple in passband
- ▶ monotonic in stopband

Design parameters:

- ▶ order N ✓
- ▶ passband max error
- ▶ cutoff frequency

Test values:

- ▶ width of transition band
- ▶ stopband error

The design parameters are the order, the pass band maximum error and the cutoff frequency.

notes

summary

2m 1s



Chebyshev lowpass

Magnitude response:

- ▶ equiripple in passband
- ▶ monotonic in stopband

Design parameters:

- ▶ order N ✓
- ▶ passband max error ✓
- ▶ cutoff frequency ✓

Test values:

- ▶ width of transition band
- ▶ stopband error

So we have one extra parameter with respect to the Butterworth. We run the algorithm and we check the result against the specs with respect to the width of the transition band and the stop band error.

notes

summary

2m 8s



Chebyshev lowpass

Magnitude response:

- ▶ equiripple in passband
- ▶ monotonic in stopband

Design parameters:

- ▶ order N
- ▶ passband max error
- ▶ cutoff frequency

Test values:

- ▶ width of transition band ✓
- ▶ stopband error ✓

If any of these parameters do not fulfill the specs, we increase the order and we run the algorithm again.

notes

summary

2m 13s



Chebyshev lowpass

Magnitude response:

- ▶ equiripple in passband
- ▶ monotonic in stopband

Design parameters:

- ▶ order N
- ▶ passband max error
- ▶ cutoff frequency

Test values:

- ▶ width of transition band ✓
- ▶ stopband error ✓

The frequency response of a Chebyshev filter looks like this.

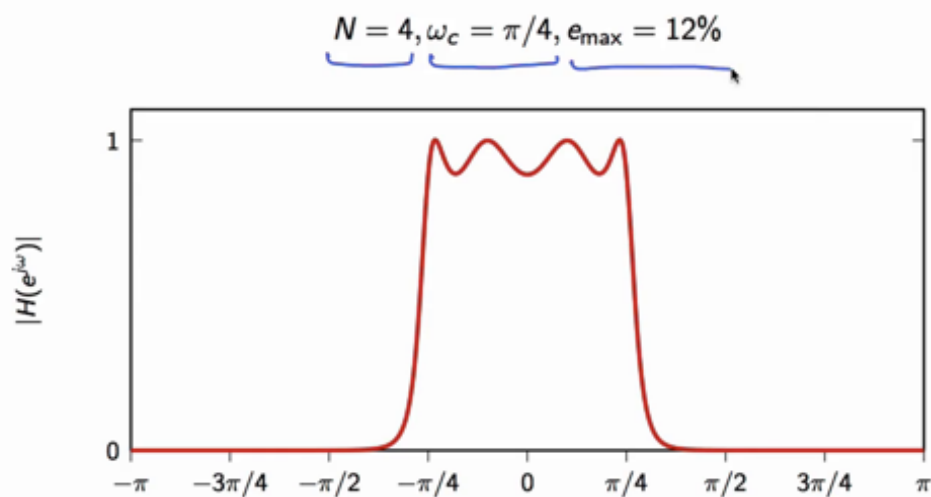
notes

summary

2m 25s



Chebyshev lowpass example



Here we have again order of four and cutoff frequency equal to π over four. And we specify a maximum pass band error of 12 percent.

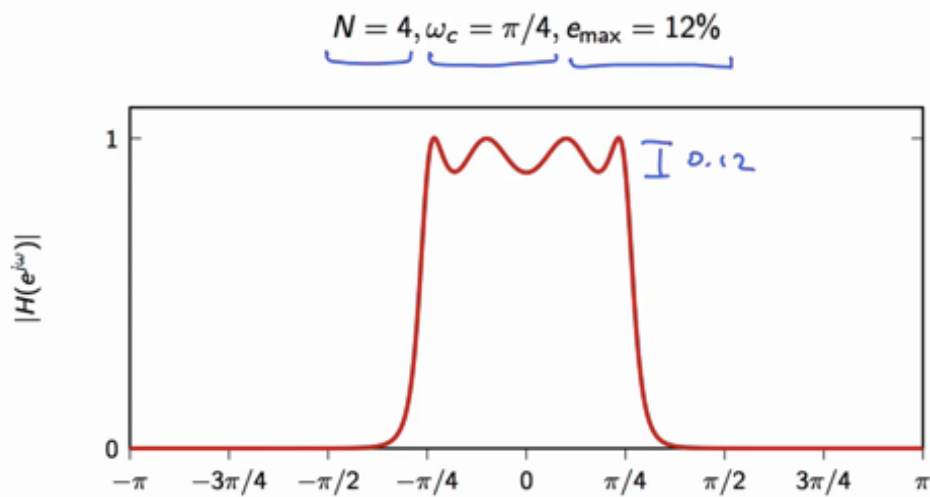
notes

summary

2m 28s



Chebyshev lowpass example



Since the desired value is one, this amplitude here is 0.12. With respect to the Butterworth filter, we have a steeper transition band and this is the reward for accepting an equiripple error in the pass band.

notes

summary

2m 38s



Elliptic lowpass

Magnitude response:

- ▶ equiripple in passband and stopband

Design parameters:

- ▶ order N
- ▶ cutoff frequency
- ▶ passband max error
- ▶ stopband min attenuation

Test value:

- ▶ width of transition band

Finally, let's look at the elliptic low pass filter. The elliptic low pass filter is equiripple both in the pass band and in the stop band.

notes

summary

2m 49s



Elliptic lowpass

Magnitude response:

- ▶ equiripple in passband and stopband

Design parameters:

- ▶ order N ✓
- ▶ cutoff frequency ✓
- ▶ passband max error
- ▶ stopband min attenuation

Test value:

- ▶ width of transition band

And the design parameters are the order, the cutoff frequency, the pass band maximum error and the stop band minimum attenuation.

notes

summary

3m 1s



Elliptic lowpass

Magnitude response:

- ▶ equiripple in passband and stopband

Design parameters:

- ▶ order N ✓
- ▶ cutoff frequency ✓
- ▶ passband max error ✓
- ▶ stopband min attenuation ✓

Test value:

- ▶ width of transition band

We run the algorithm and we check the result in terms of the width of the transition band.

notes

summary

3m 10s



Elliptic lowpass

Magnitude response:

- ▶ equiripple in passband and stopband

Design parameters:

- ▶ order N
- ▶ cutoff frequency
- ▶ passband max error
- ▶ stopband min attenuation

Test value:

- ▶ width of transition band

So you see this design lets you control all the parameters of the filter except the transition band. So it's the most complex of the three. The frequency response looks like this.

notes

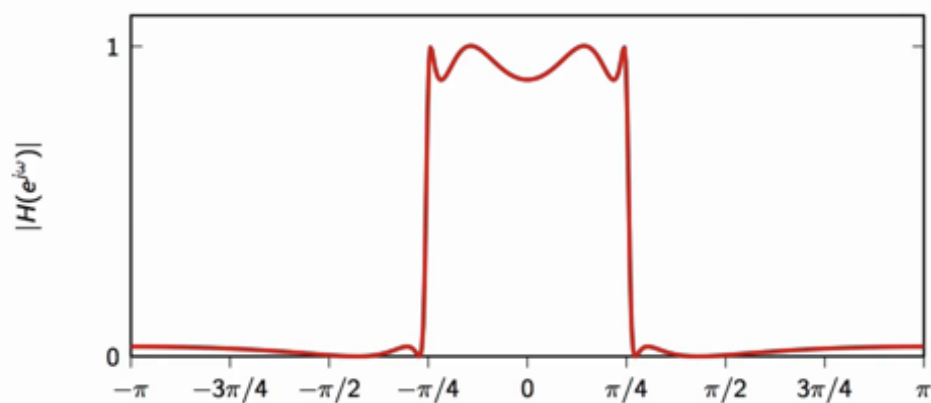
summary

3m 13s



Elliptic lowpass example

$$N = 4, \omega_c = \pi/4, e_{\max} = 12\%, \text{att}_{\min} = 0.03$$



And again our order is four. The cutoff frequency is π over four. And here we specify a maximum pass band error of 12 percent and a minimum attenuation of 0.03.

notes

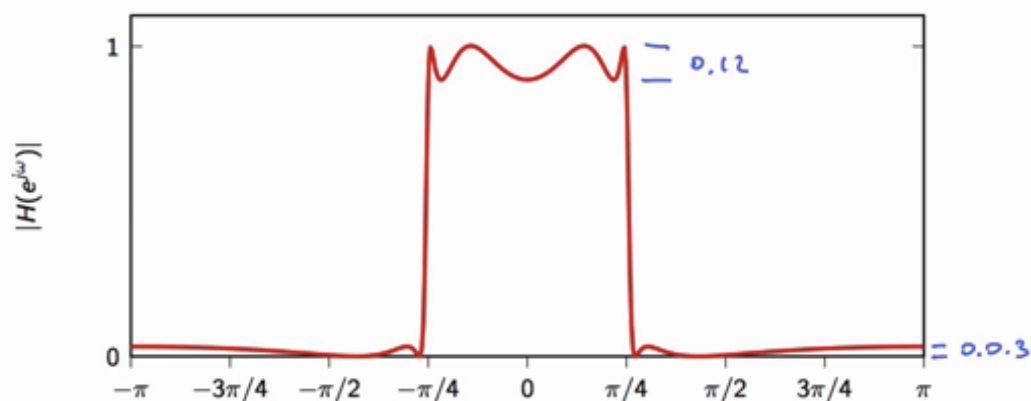
summary

3m 26s



Elliptic lowpass example

$$N = 4, \omega_c = \pi/4, e_{\max} = 12\%, \text{att}_{\min} = 0.03$$



So this is 0.03 and this is 0.12. The elliptic filter gives you the steepest transition band for a given order of all the filters we have seen so far.

notes

summary

3m 39s

