

Course material

Course:

Digital Signal Processing II

Video:

2.2.6 SOTD Image Resolution and Space Exploration

Concepts (extracted from automatically generated subtitles):

Camera resolution. Original light sources. Closest points. Very small aperture. Pinhole camera. Big telescope. Space exploration. Sensor of a camera. Mechanics of image formation. Light travels. Points of interest. Diffraction pattern. Point spread function. Smallest aperture. Angular separation.



[to video sequence search](#)
(within Digital Signal Processing II.)

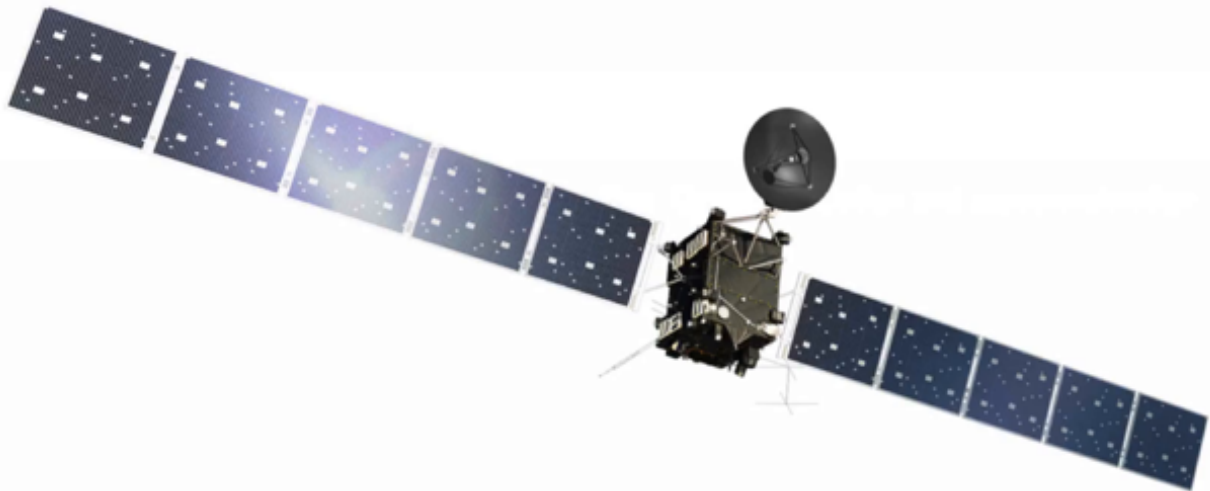


[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

The Rosetta spacecraft



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Hi and welcome to today's Signal of the Day in which we will talk about camera resolution and space exploration. We will start our discussion on camera resolution by talking about one of the most exciting scientific missions to take place in the last ten years, the Rosetta mission. The Rosetta spacecraft was launched about ten years ago by the European Space Agency with the mission of reaching the comet Turium of Gerazimenko. This comet is also known with the more technical but easier to pronounce name 67P. Rosetta had to fly through space for ten years in order to gain the necessary momentum to join the comet on its orbit.

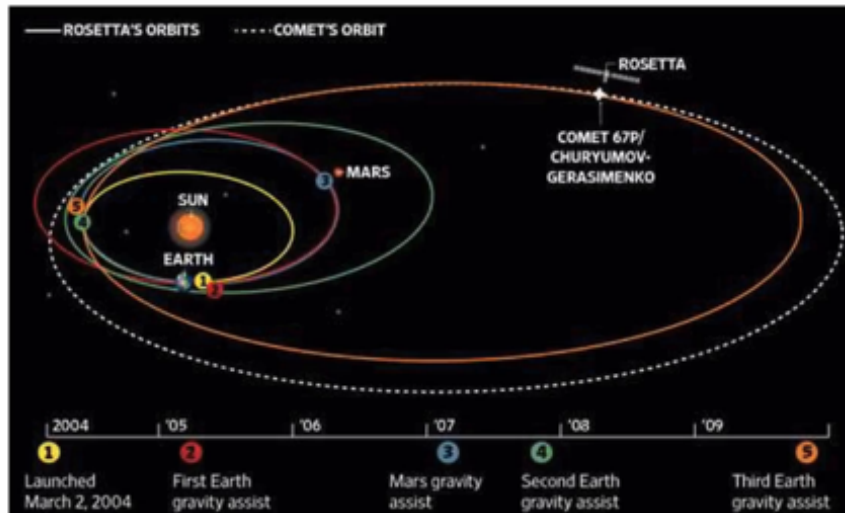
notes

summary

0m 0s



Rosetta's flight path



Source: NASA, WSJ

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And it had to fly by Mars and Earth over four times to get what is called a gravitational assist. Basically, exploiting the gravitational field of planets to gain speed and reach the comet.

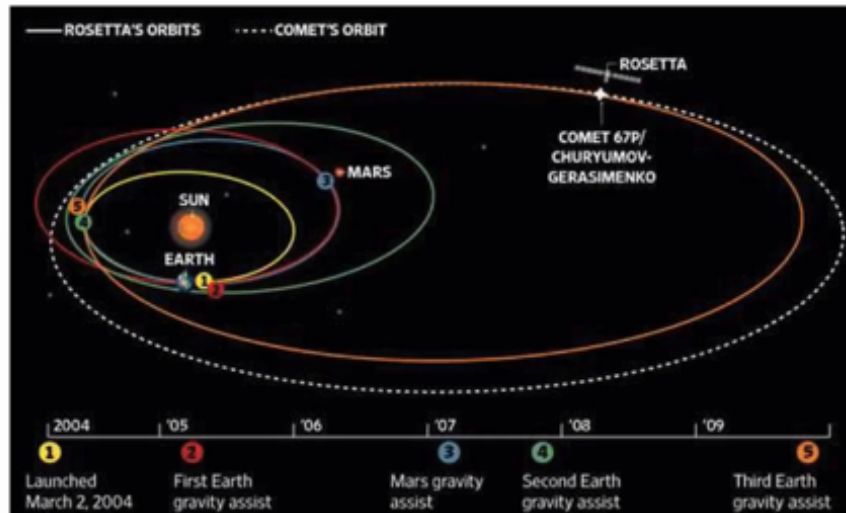
notes

summary

0m 38s



Rosetta's flight path



Source: NASA, WSJ

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The total flight path was over 6.4 billion kilometers and Rosetta successfully reached the comet 67P in August 2014.

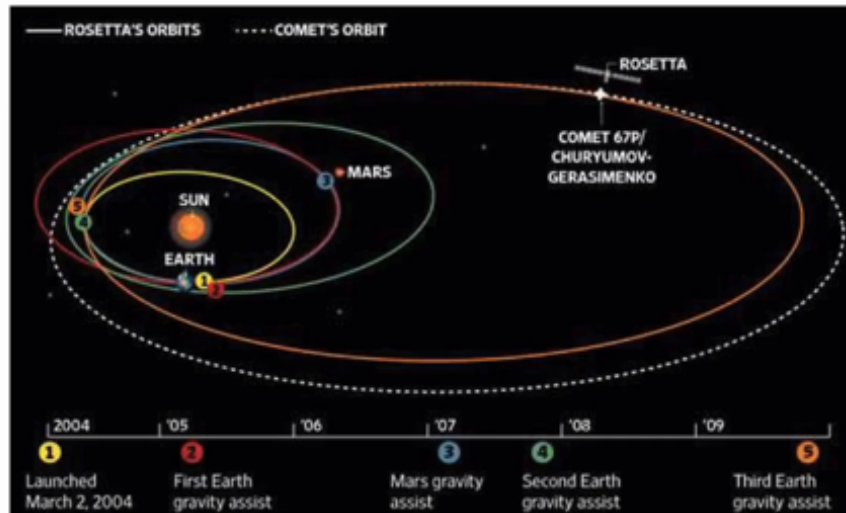
notes

summary

0m 50s



Rosetta's flight path



Source: NASA, WSJ

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And it became actually the first man-made object to orbit a comet. Rosetta also carried a lander along called Philae, which was supposed to descend on the comet and analyzed its physical and chemical properties.

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summary

1m 8s





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But unfortunately not everything went perfectly well with the lander and only time will tell if the lander will be able to complete its part of the mission. But if we go back to the main topic of our video today, what Rosetta managed to do was to send back to Earth increasingly detailed pictures of the comet as it was getting closer to it. So here's the first picture from 86,000 kilometers away and the underlying resolution is one kilometer per pixel. As Rosetta got closer to the comet, the quality of the pictures progressively improved and here we start seeing the shape of the comet for instance. From 5,000 kilometers we have a resolution of 100 meters per pixel, from 285 kilometers the resolution is now 5.3 meters per pixel and finally one of the closest points that Rosetta managed to reach once it entered orbit around the comet is a distance of 6 kilometers for a resolution of 11 centimeters per pixel. These are the most detailed pictures ever taken of a comet. But the question is why do we have to send a probe for 10 years into space in order to get higher resolution pictures of the comet? Can't we just point a big telescope and obtain the same result? And this is the question that we will try to answer today. People have been looking at the sky since time immemorial and it is a fact of life that you don't really expect to see a lot of detail when you look up at the sky with the naked eye. It has always been accepted that the limiting device in appreciating the complexity of the cosmos is the human eye.

notes

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1m 15s





notes

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$$i(x, y) = s(x, y) * h(x, y)$$

$$= s(x, y) * \underbrace{t(x, y)} * \underbrace{p(x, y)}_{\text{PSF}}$$

whether it's a camera or whether it's the eye, that we don't really have to take their distance into account. If we accept this model then the image that is formed, say on the retina or on the sensor of a camera, is the convolution between the original light sources and a transfer function that accounts for all the transformations that light undergoes from the source to the imaging plane. It is convenient to split the transfer function into two distinct components. The first one describes the properties of the medium through which light travels, whereas the second one is dependent on the imaging device and takes into account the effects of say lenses and focal distance and characteristic of the imaging plane. This is also called the point spread function and as we will see shortly it characterizes the properties of an imaging device. The reason why we perform the split is because the major enemy to image quality for telescopes on Earth are the disturbances introduced by the atmosphere.

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3m 31s





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Atmospheric disturbances include turbulence and scattering and haze and you probably have experienced those even with your naked eye when you see the sun close to the horizon. The distortion of the shape of the sun is due to the fact that the sun rays have to travel through a thicker slice of the atmosphere at sunset before reaching your eyes. Of course today we are particularly privileged because in 1990 NASA put into orbit the first space telescope, Hubble, and so today we can perform telescopic observations that are immune from the effects of the atmosphere. But even with the space telescope our vision is limited. We will see why in a second, but for the time being as a point of reference, here are the best pictures of the comet P67 as seen by the Hubble telescope. And you can see there are far cry from the kind of images we got from Rosetta. So with the atmosphere out of the way we can concentrate on the mechanics of image formation and on the point spread function associated to the imaging device. So let's set up a very simple model where we have a point light source, so it's a star, say a far away star, that is sending light rays in all directions. Instead of a telescope let's put somewhere a very simple type of camera, a pinhole camera. A pinhole camera is a very simple device, it's just a box with a very small aperture here, the smallest aperture you can imagine, and an imaging sensor on the back plate here. It could be film or it could be a digital CCD. The properties of the pinhole camera are such that from every

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summary

4m 37s





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light point source in space only one ray of light can actually go through the aperture and reach the back plate. So that for each light source in space you have one imaged point on the image sensor. If you have two light sources, well it's easy to find geometrically which ray ends up entering the camera through the pinhole, and for each point in space you will have the corresponding point on the image plane. From this simple geometrical model it would appear that the ideal pinhole camera has a point spread function which is just a delta function, namely nothing happens to light in the imaging process. In other words, the pinhole camera appears to have infinite resolution, and so we might be tempted to say that the maximum achievable resolution of a system is only dependent on the density of the sensor, the digital sensor that is put in the back plane of the camera. In other words, if we assume that the back plane of the pinhole camera is composed of an array of light sensitive elements, let's call them pixels, like so, we can see that in this picture where we have indicated with different colors successive pixels, if the angular separation between points in space is too small with respect to the width of the pixel, two different points in space might end up mapped to the same pixel, and therefore we wouldn't be able to tell them apart in the resulting image. So in order to resolve two separate light sources in space, we have to make sure that the density of pixels is such that each ray that goes through the pinhole falls on a different pixel on the camera, and

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furthermore we need to have an intermediate pixel between the lit pixels to make sure that we can distinguish between light, dark, and light, and recognize the picture as two separate sources. Alternatively, if we can't afford a higher pixel density, what we can do is build a longer pinhole camera. This is equivalent to using a zoom lens in a normal camera, so you see that by extending the body of the camera, we can use a lower pixel density and still achieve resolution with respect to the same angular separation of the light sources. However, in reality, resolution is not limited by pixel density. Resolution is limited by diffraction, and this is when things start to get a little complicated. When light goes through an opening, like in the case of the pinhole camera, it starts exhibiting a wave-like nature, and the image of the original point source will actually appear as a diffraction pattern. The diffraction pattern through a small circular aperture is called the airy disc, and here we're showing how it looks. Actually, we're showing the negative image of the diffraction pattern of light for better contrast. What this means is that on the back plane of the camera, whether the camera is digital or analog, instead of seeing a single point corresponding to the original point source, you would see this diffraction pattern. Now, when you have two point sources, each source will generate its own independent airy pattern, and so instead of having two points here and here, you will have two diffraction patterns that will start to interact when the angular separation between sources is small. The patterns will ultimately overlap so much that you will not be

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able to resolve the individual sources independently of the sensor. And so here, for instance, you have two airy patterns that are overlapping into one undistinguishable blob. So you see, independently on the sensor of the camera, there is an angular resolution between light point sources as seen by the camera, under which no resolution is possible. The minimum angle that guarantees resolution is given by Rayleigh's criterion, and the formula is like so. It will depend on the wavelength of the light that hits the camera and on the diameter of the aperture. It's actually inversely proportional to the diameter of the aperture, so the smaller the aperture, the bigger the airy disk. And so a pinhole camera would be an extremely bad device to obtain sufficient angular resolution for very distant objects. Okay then, if aperture is the problem, let's increase the aperture and therefore increase the angular resolution of the camera. Now, increasing the aperture is not as simple as drilling a bigger hole in the pinhole camera, because if we do so, then each light source will have a cone of rays of light that will enter the camera. This cone of rays of light will diverge inside the camera and create an undifferentiated blob of light on the back plane. So what we need to do is put a lens on the aperture so that the cone of rays that hits the lens will be focused back on the back plane, and one point source will be mapped onto one point on the sensor plane. Similarly, when we have two point sources, the lens will focus the rays again on a different point, and we have the same situation as

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The Lunar Landing Module



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before, but with a larger aperture for the camera. This incidentally is also very good because it means that more light enters the camera, and so the exposure time can be shorter, which is very good for moving objects. However, now our system is dependent on an extra piece of hardware, namely the lens. And let me tell you right away that making precise lenses is a very, very difficult task, which will limit the kind of apertures that we can use in real life. But before we even consider that problem, let's see what kind of aperture the Hubble telescope should have in order to be able to see the lunar landing module. As you may have heard, there are some people out there, especially on the Internet, that claim that the Apollo missions were a hoax and that man never landed on the moon. And what they say is, if indeed man had landed on the moon, we should be able to point a telescope to the moon and see the leftovers from the moon missions, such as the lunar landing module. So let's do the math. The size of the lamp is about 5 meters across,

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Seeing the Lunar Excursion Module (LEM)



- ▶ size of LEM $\approx 5\text{m}$
- ▶ distance to the Moon $\approx 382,000\text{km}$

$$\theta \approx 0.003 \text{ arcsec}$$

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and the distance from Earth or the Hubble telescope to the moon is about 380,000 km. If you do the trigonometry, you will see that the angle subtended by the lamp is about 0.003 arc seconds, and 1 arc second is 1 degree divided by 3600. So this is an infinitesimally small angle. The Hubble telescope has an aperture of 2.4 meters.

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summary

12m 46s



Seeing the Lunar Excursion Module (LEM)



- ▶ Hubble's aperture: 2.4m
- ▶ visible spectrum from 400nm to 700nm; pick $\lambda \approx 550\text{nm}$
- ▶ Rayleigh's criterion: $\theta \approx 0.1 \text{ arcsec}$

to see the LEM, Hubble should have an aperture of 80m!!!

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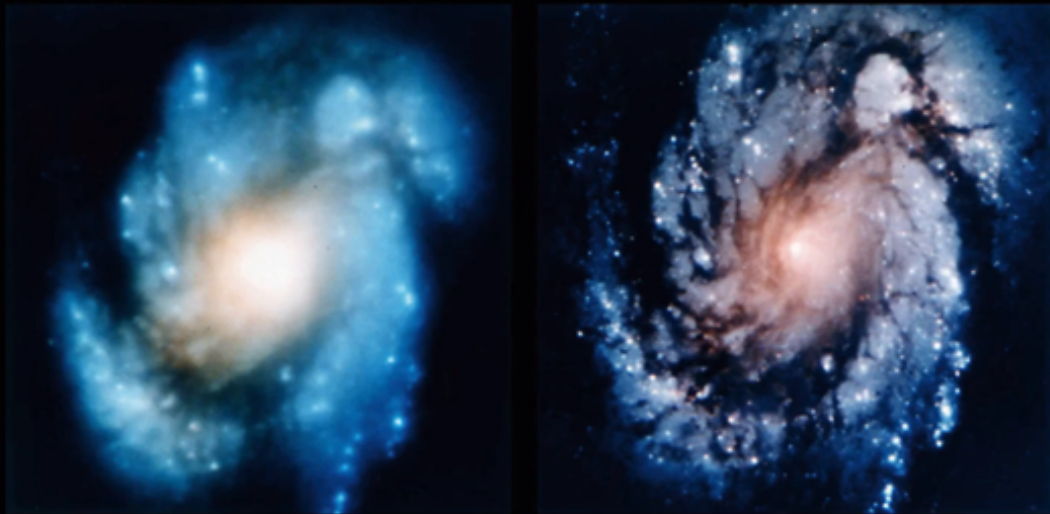
To compute the angular resolution of such an aperture, remember Rayleigh's criterion requires us to specify the wavelength of the incident light. Now, the visible spectrum is from 400 nanometers to 700 nanometers. Let's pick 550 nanometers as the midpoint. We plug these values into Rayleigh's criterion, and we get an angular resolution for the Hubble telescope of 0.1 arc seconds. Proportionally, if we were to see the lamp on the moon, the Hubble should have an aperture of 80 meters. Now, even if we could put in space something as big as that, we would still have to build a lens 80 meters across with perfect focus. And this is currently way beyond the limits of our technology. Any small imperfections in the curvature of the lens would result in an out-of-focus and blurred image.

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13m 14s





an error of $2.2\mu\text{m}$ for a 2.4m-wide mirror!

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This is really what happens if you're short-sighted, for instance. Your lens is too round for the length of your eye, or your eye is too long for the thickness of your lens, and the focus will happen before the retina. To give you an idea of the level of precision this is required by large aperture optics, consider that when the Hubble telescope was launched in 1990, astronomers realized that there was a minute error in the calibration of the optics. The Hubble telescope, like all large telescopes, doesn't use lenses to focus the image, but uses mirrors. Now, there's a mirror in the back of the Hubble telescope that has the same diameter as the aperture of the telescope, so we're talking about a 2.5-meter-wide mirror. The curvature of the mirror had an imprecision of 2.2 micrometers at its edge, and as a result, Hubble could not focus properly. In the left-hand side of this picture, you see how a galaxy would look before corrective optics were installed on the telescope. And on the right-hand side of the picture, you see the picture after correction. So you see a minute error of 2.2 micrometers over 2.5 meters of diameter was already enough to cause this kind of blurriness. Okay, but so what about pixels? How does the camera resolution impact image quality?

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14m 7s



$$\Delta x = 1.22\lambda \frac{f}{D}$$

f/D : f-number

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To understand that, we have to go back to Rayleigh's criterion for the minimum resolvable angle in a camera. That angle maps to an actual physical distance on the camera sensor via this formula here. And if you look at it in detail, you see that this is equal to theta,

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summary

15m 32s



$$\Delta x = 1.22 \lambda \frac{f}{D} = \theta \cdot f$$

f/D : f-number

32

which is the minimal angle given by Rayleigh's criterion, times the focal length. What this formula tells us is that if the pixel separation on the camera sensor is not less than Δx , our camera will be resolution-limited rather than diffraction-limited.

notes

summary

15m 52s



$$\Delta x = 1.22 \lambda \frac{f}{D} = \theta \cdot f$$

f/D : f-number

Note that f over d , which appears in the formula here, is what photographers call the f-number of a camera. So imagine you have two point light sources that have an angular separation that is just resolvable according to Rayleigh's criterion.

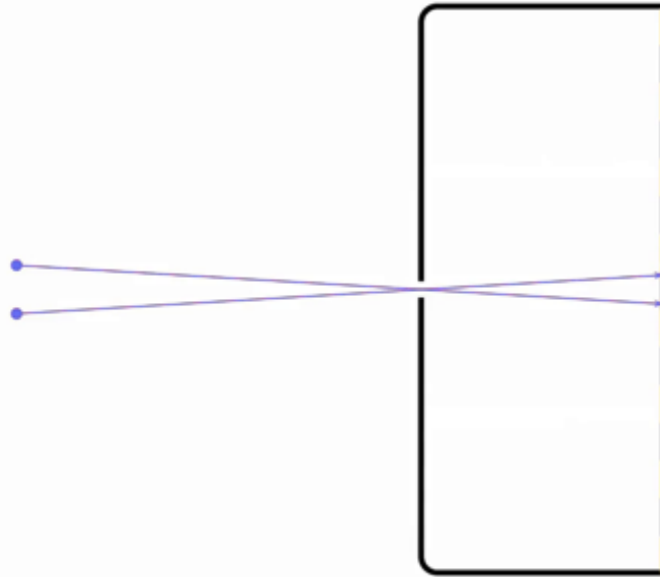
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16m 8s



Number of pixels vs pixel density



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On the camera's sensor, you would observe the two diffraction patterns relative to the two light sources. And in order to be able to distinguish the two light sources, your pixel size would have to be small enough to capture the light pattern here, the darker pattern here, and the light pattern here. If the pixels are any bigger, then we will not be able to resolve the light sources due to a lack of resolution. It is clear from this discussion that what is important in a camera sensor is not so much the absolute number of pixels, but the pixel density. So for instance, this sensor here would have the same number of pixels as this sensor here, but clearly this sensor will not be able to resolve the same two point light sources,

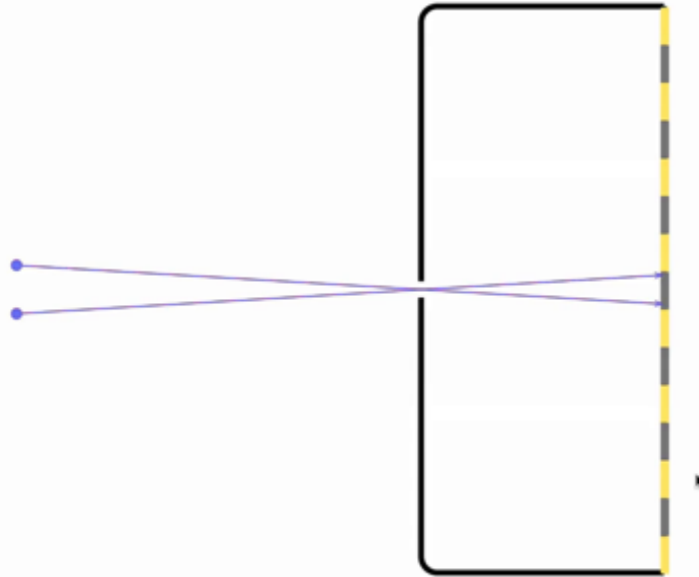
notes

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16m 18s



Number of pixels vs pixel density



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because the pixel density in this sensor is much lower.

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summary

17m 11s



What about megapixels?



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When it comes to commercial cameras, the marketing slogans are about the number of pixels, but hardly ever about the size of the sensor. So how many megapixels do you actually need in your camera? This really depends on the size of the sensor and on the optics.

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17m 20s



What about megapixels?



- ▶ f-number of all trades: $f/8$
- ▶ using the spatial Rayleigh criterion: $\Delta x \approx 4\mu\text{m}$
- ▶ max pixel area $16 \cdot 10^{-5} \text{ mm}^2$

to operate at the diffraction limit we need 62,500 pixels/mm²

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So the minimal resolution, the minimal pixel density, will depend on the f-number as we showed just before.

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summary

17m 35s



What about megapixels?



- ▶ f-number of all trades: $f/8$
- ▶ using the spatial Rayleigh criterion: $\Delta x \approx 4\mu\text{m}$
- ▶ max pixel area $16 \cdot 10^{-5} \text{ mm}^2$

to operate at the diffraction limit we need 62,500 pixels/mm²

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So let's pick an f-number that works pretty much in most conditions, f over 8. If we use the spatial Rayleigh criterion, we find that the pixel pitch, the width of the pixel, that we need to be diffraction limited rather than resolution limited, will have to be 4 micrometers. So that means that the maximum pixel area will be $16 \times 10^{-5} \text{ mm}^2$. Or if we flip the perspective, it means that to operate at the diffraction limit, we need approximately 62,500 pixels per square millimeter. So now we need to look at the size of the sensor.

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summary

17m 38s



What about megapixels?



- ▶ f-number of all trades: $f/8$
- ▶ using the spatial Rayleigh criterion: $\Delta x \approx 4\mu\text{m}$
- ▶ max pixel area $16 \cdot 10^{-5} \text{ mm}^2$

to operate at the diffraction limit we need 62,500 pixels/mm²

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High-end cameras today usually have one of these two sensor formats.

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summary

18m 24s



What about megapixels?



- ▶ APS-C sensor (329mm^2): 20 MP
- ▶ 35-mm sensor (864mm^2): 54 MP

don't buy more megapixels than you need ;)

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The APS-C, which is very common, and has an area of 329 mm^2 . With that sensor, after you reach 20 megapixels, your camera is already operating at the diffraction limit. So no use in increasing the number of pixels. For a full sensor that simulates 35 mm film and has an area of 864 mm^2 , the limit will be 54 megapixels. So in the end, don't buy more megapixels than you need. Now let's go back to our Rosetta spacecraft.

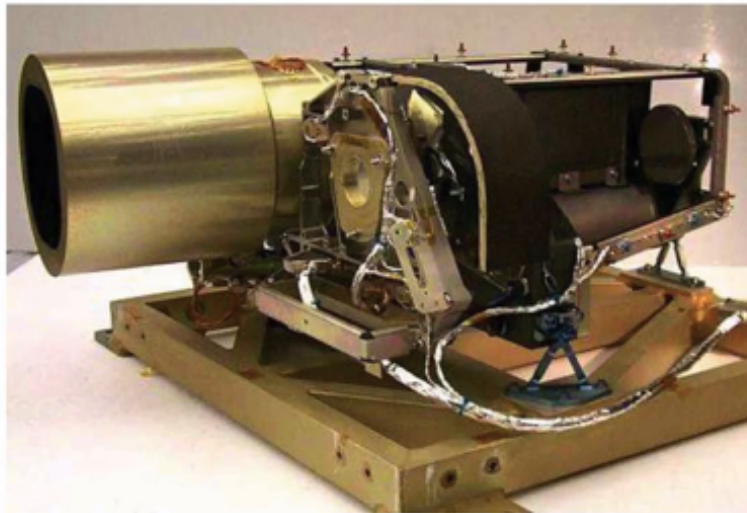
notes

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18m 25s



What about Rosetta?



4MP sensor

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This is the camera that is on board and that has sent us back these beautiful pictures of the comet. And you'll be surprised to know that the resolution of the camera is only about 4 megapixels. So the moral of the story is, you don't need to buy a camera with a lot of pixels if you're willing to travel 6 billion kilometers to get close to your target.

notes

summary

18m 59s

