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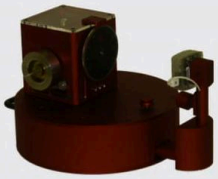


$G_\omega(s)$

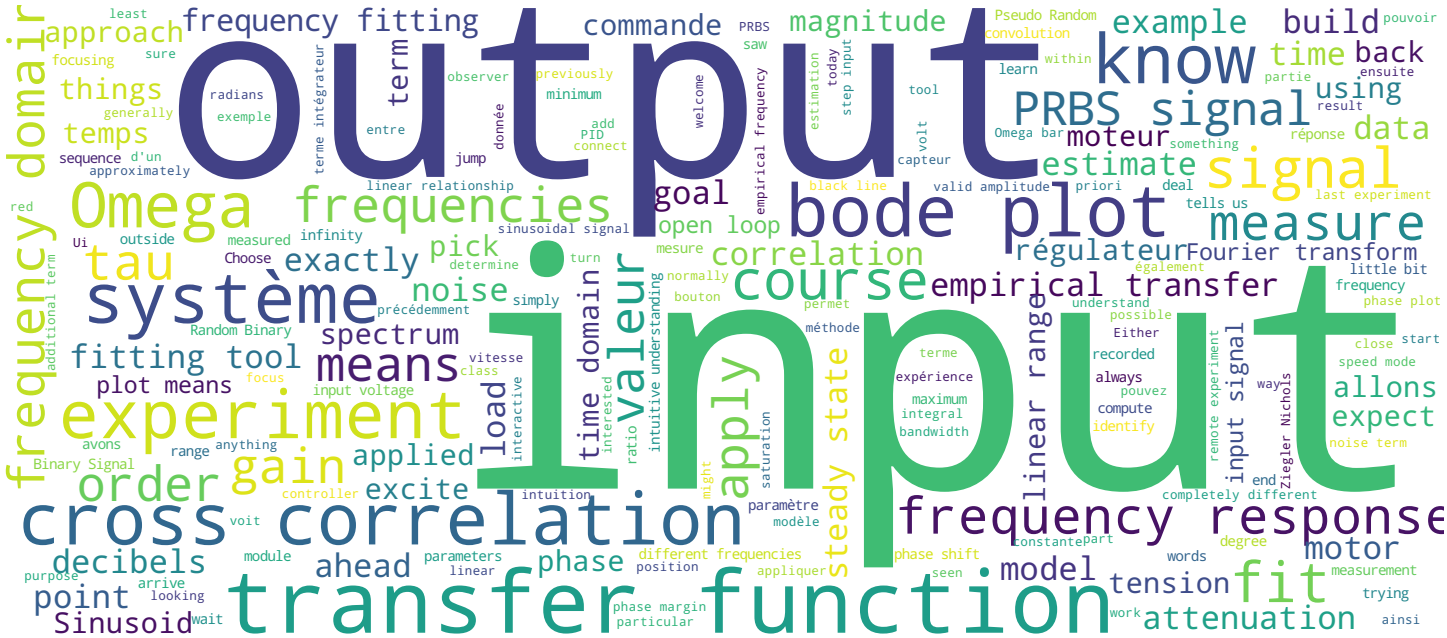
System Modeling

Experimental model identification in the frequency domain

Control Systems' Hands-on Sessions

Christophe Salzmann & Colin Jones

Automatic Control Lab



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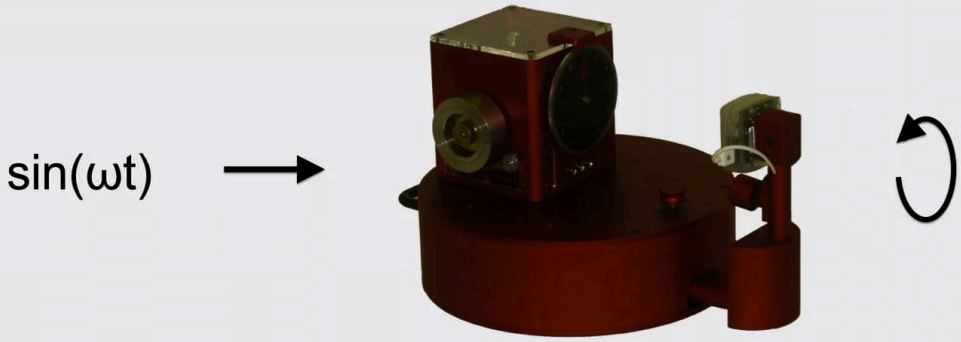


What is the dynamic relationship between input [volt] and output speed [volt]?
This time in the frequency domain via PRBS signals and sinusoids

TP Control Systems

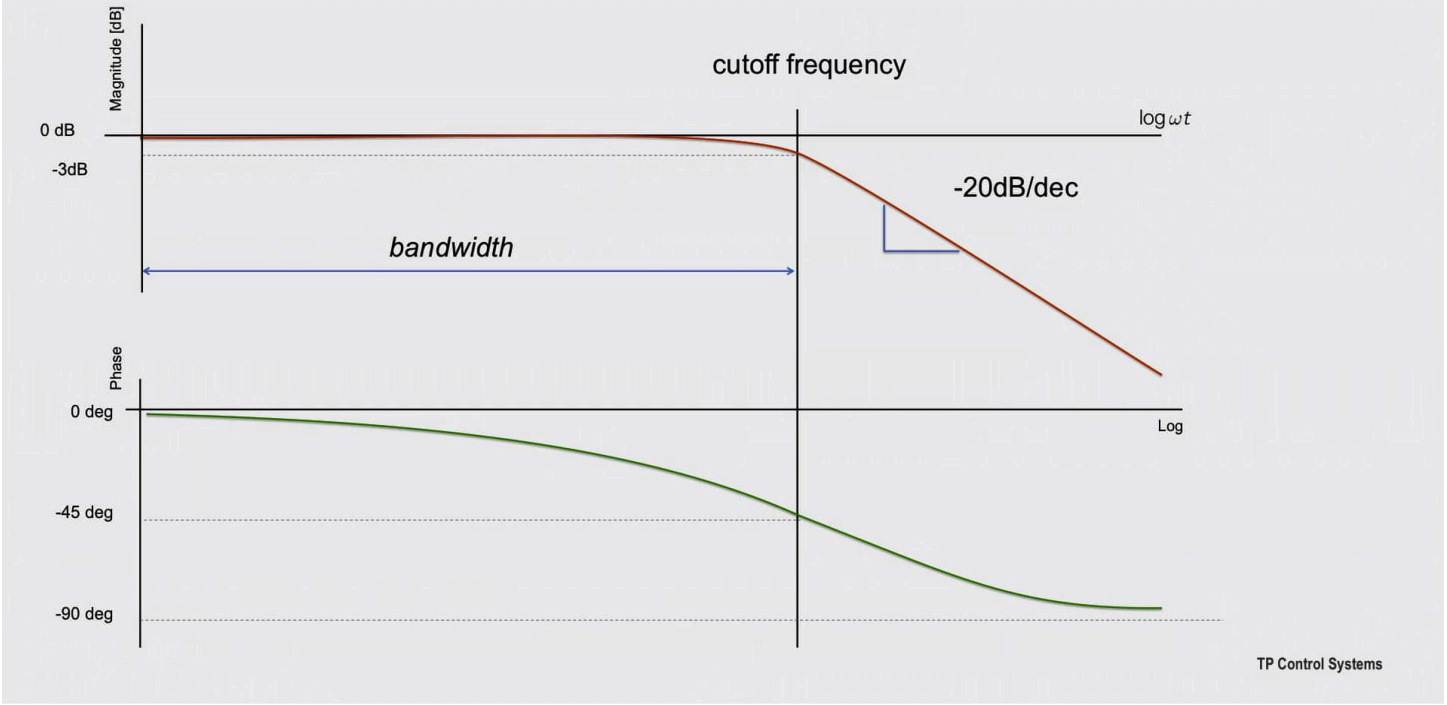
Hello and welcome to your second experiment on system modelling. Today we're going to focus on experimental model identification, but this time we're going to do it in the frequency domain instead of in the time domain. Our goal today is to build a model of the dynamic relationship between our input voltage that we apply to the motor and the output speed in the motor. We want to learn the transfer function from input to output, and this time we're going to do it in the frequency domain. We're going to do this in two ways. First, we're going to apply a PRBS signal, which is a Pseudo-Random Binary Signal to our system, and then from this build our transfer function. The second approach, we're going to apply Sinusoids at very specific frequencies and then measure what the output is at those points.

Notes

Summary

0m 10s

Bode plot



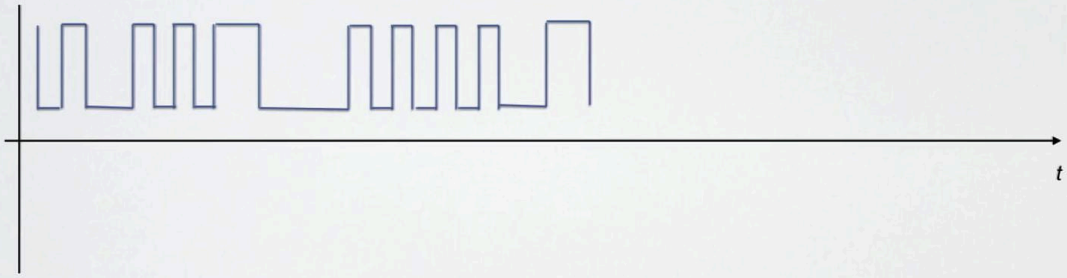
We have two goals. The first is to understand how we can construct bode plot from experimental data that we measure from our system. The second is to develop more of an intuitive understanding of what a bode plot actually means. If I give you a bode plot of a particular shape, how do we expect this system to behave? These are the two things that we should be focusing on when we're doing the exercise today.

Notes

Summary

0m 40s

PRBS signal



Excite many frequencies at the same time

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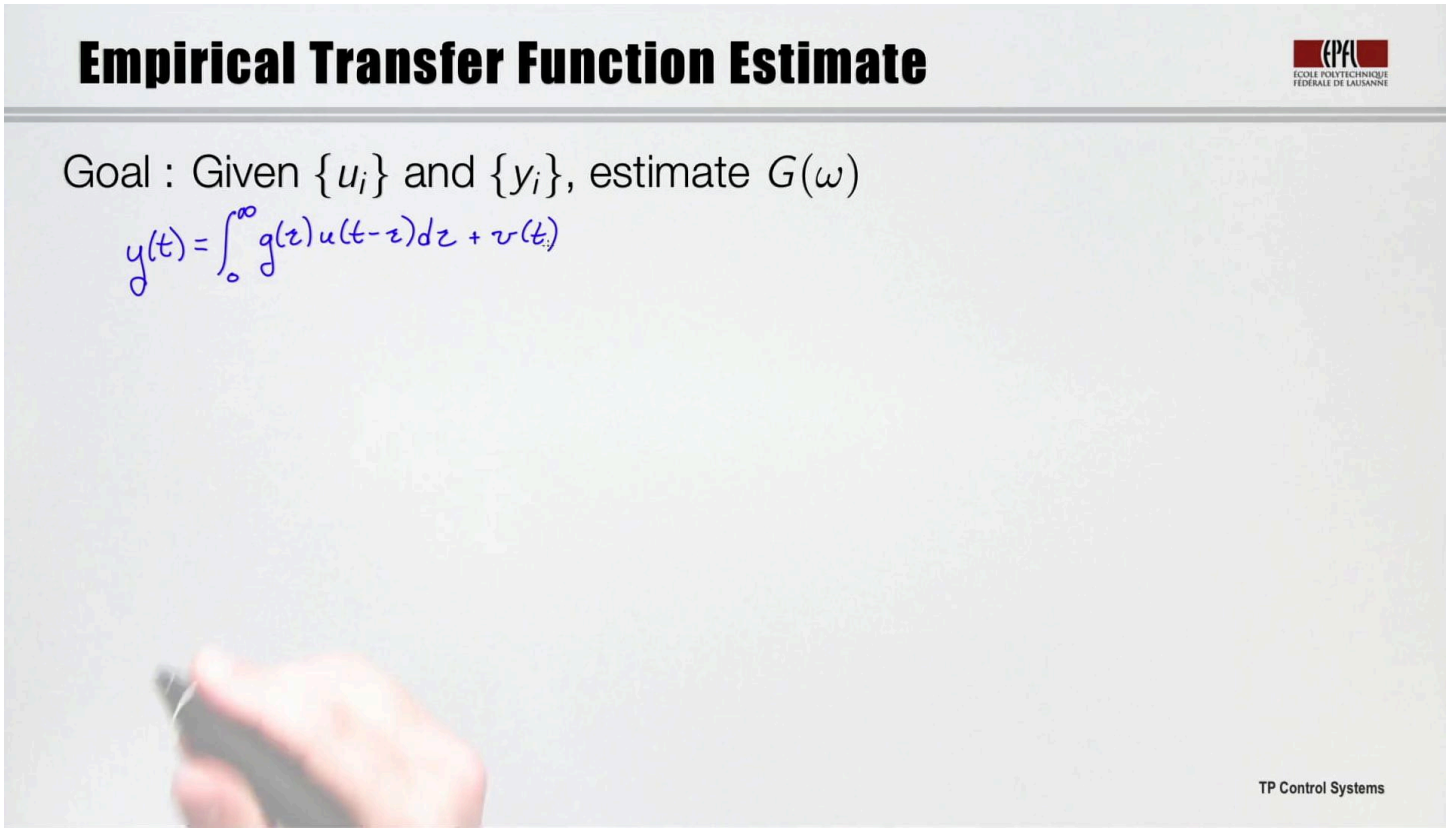
In the first experiment, what we're going to do is to apply a PRBS or Pseudo-Random Binary Signal to our system. You can see an example of a PRBS signal over here. This is a signal that jumps from a minimum to a maximum, up and down in a random fashion. What we know is that a jump like this, a step input like this, excites all the frequencies of our system. In other words, a PRBS signal is actually just a signal that is exciting all of the frequencies of our system. Therefore the output should contain all of the frequency content of our system, which should allow us to build a frequency response or a transfer function that we're looking for.

Notes

Summary

1m 04s

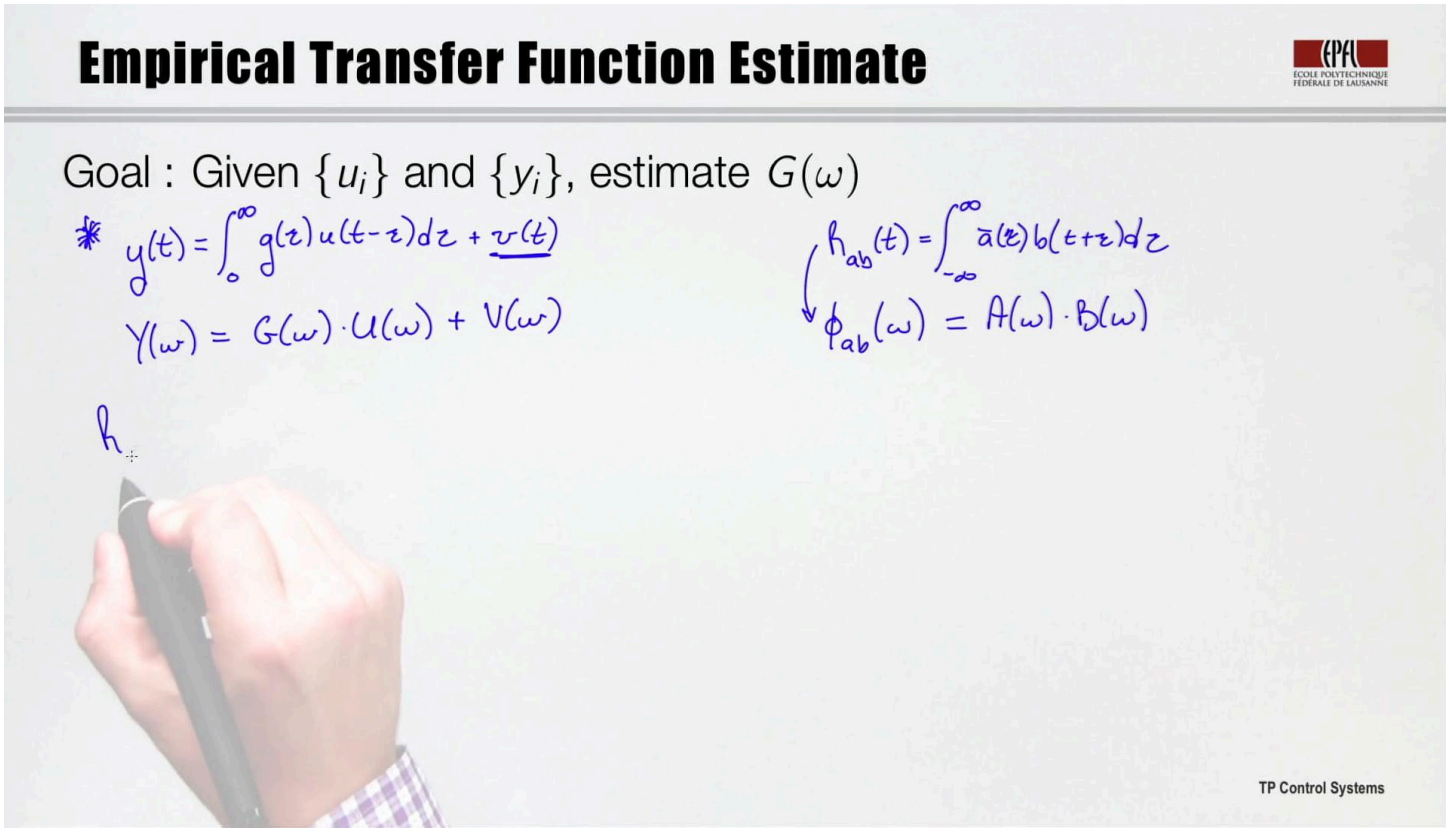


At the end of this experiment, what we'll have is a sequence of inputs that we've applied to the system Ui as well as a sequence of outputs that we measure from the system yi. Our goal is to estimate our transfer function G of Omega. We, normally, of course, write this as G(s) in the Laplace domain, whereas here G of Omega we can take to be G of j Omega just for shortening notation. Okay, so what we want to do is to ask how we can get from these inputs and outputs to this transfer function G of Omega. Or what we do is we start from the linear relationship that we have between inputs and outputs. We know our output at time t, y of t, is just the convolution of our impulse response g of tau and our input u of t minus tau d tau. This is what we write in class. But of course, this is the real world and so there's always one more term that we have to add. There's always some additional term v of t, which is noise. This is a term that... This could be measurement noise, could be modelling errors, could be anything, but we simply don't know what it is. But we have to deal with this when we're doing a real experiment. We want the frequency response of this system.

Notes

Summary

1m 40s

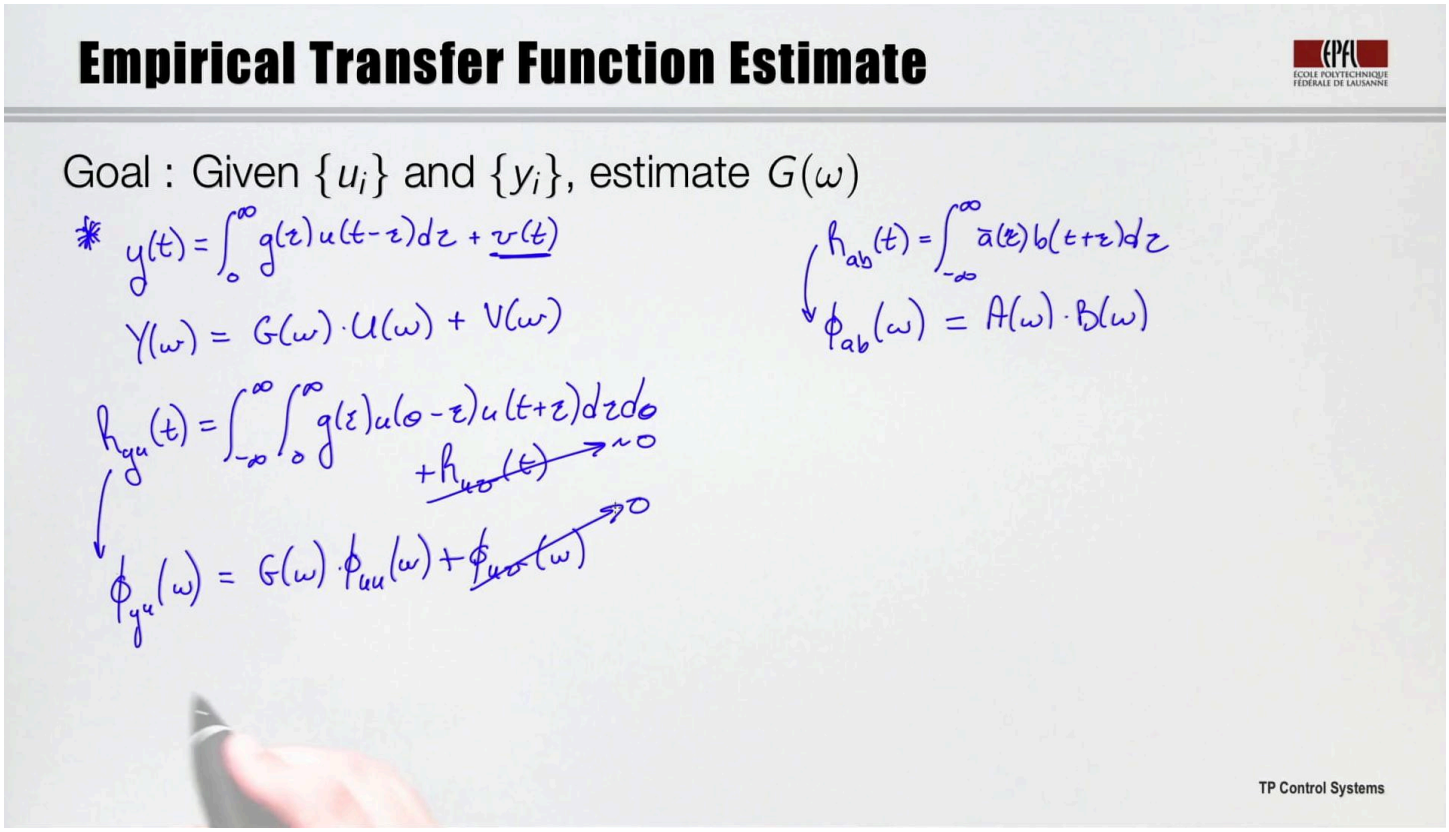


The first thing we can do is we can take the Laplace transform or equivalent to the Fourier transform of our system, which gives us a linear response or a linear relationship between our outputs and our inputs with our transfer function sitting here, G of Omega. But of course, we also have this noise term, so we have to account for that in the frequency domain as well. The problem here is we know Y, we know U, and we want G, but we don't know V. We don't have enough data or enough relationships in order to determine what our transfer function G of Omega actually is. How do we deal with this? The first thing we do is we define the cross-correlation between two signals, a and b. The cross-correlation r sub ab, we justify as the integral from minus infinity to infinity of the complex conjugate of a of tau b of t plus tau d tau. In other words, it's just the convolution of a and b. We can now take the Fourier transform of our cross-correlation... Which, because this is simply convolution, we know is just going to be multiplication in the frequency domain. Now that we have this, what we can do is we can go back to our initial relationship for our system and we can take the cross-correlation with what we've measured our output y of t with the input that we applied to our system.

Notes

Summary

2m 53s



Which looks messy, but it's just simply another convolution added into the mix. Of course, we have one additional term, which is the convolution of our noise term v with our input u. This tells us why we take the cross-correlation with the input of our system. This is because the input is completely uncorrelated with the noise of our system. The input was chosen randomly. The noise is a completely different random signal. When you take the cross-correlation of these two things, you expect to see zero. It's because the cross-correlation is a measurement of the similarity between two signals. And so we would expect this cross-correlation to be very, very close to zero. Now if we take the Fourier transform of this, what we get in the frequency domain is... What we get in the frequency domain is the spectrum of the correlation or the cross-correlation between our output and our input. Our transfer function G of Omega over here and then the cross-correlation of the input with itself, which is just the autocorrelation. Then finally over here, the spectrum of this term, which is the correlation or the cross-correlation between our input and our noise, which we know to be almost zero.

Notes

Summary

4m 46s

Empirical Transfer Function Estimate

Goal : Given $\{u_i\}$ and $\{y_i\}$, estimate $G(\omega)$

$$y(t) = \int_0^\infty g(\tau)u(t-\tau)d\tau + v(t)$$

$$Y(\omega) = G(\omega)\cdot U(\omega) + V(\omega)$$

$$h_{ab}(t) = \int_{-\infty}^{\infty} a(\tau)b(t+\tau)d\tau$$

$$\phi_{ab}(\omega) = A(\omega)\cdot B(\omega)$$

$$h_{yu}(t) = \int_{-\infty}^{\infty}\int_0^\infty g(\tau)u(\theta-\tau)u(t+\tau)d\tau d\theta + h_{uv}(t) \to \sim 0$$

$$\phi_{yu}(\omega) = G(\omega)\cdot\phi_{uu}(\omega) + \phi_{uv}(\omega) \to 0$$

$$G(\omega) \approx \frac{\phi_{yu}(\omega)}{\phi_{uu}(\omega)}$$

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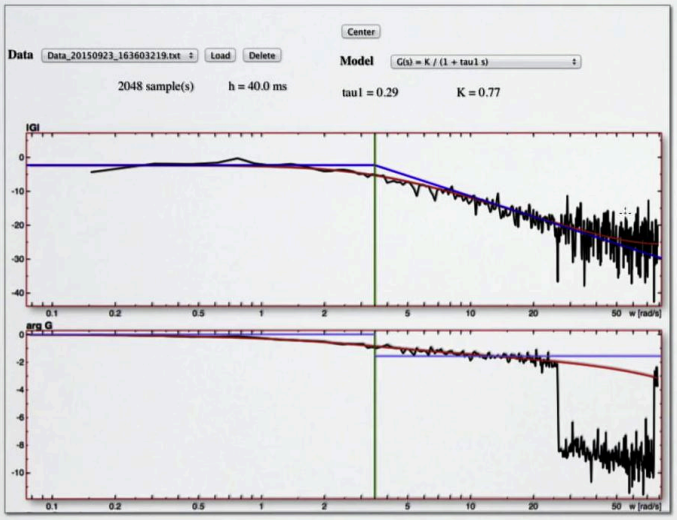
Now what we can do, we want to estimate this transfer function so we can write that our transfer function G of Omega is approximately equal to the spectrum of the correlation between output and our input, divided by correlation between our input with itself. This is what we call the empirical transfer function estimate, and it's an estimation of the frequency response of our system.

Notes

Summary

6m 25s

Frequency-fit tool



Remote experimentation tool computes the empirical transfer function estimate

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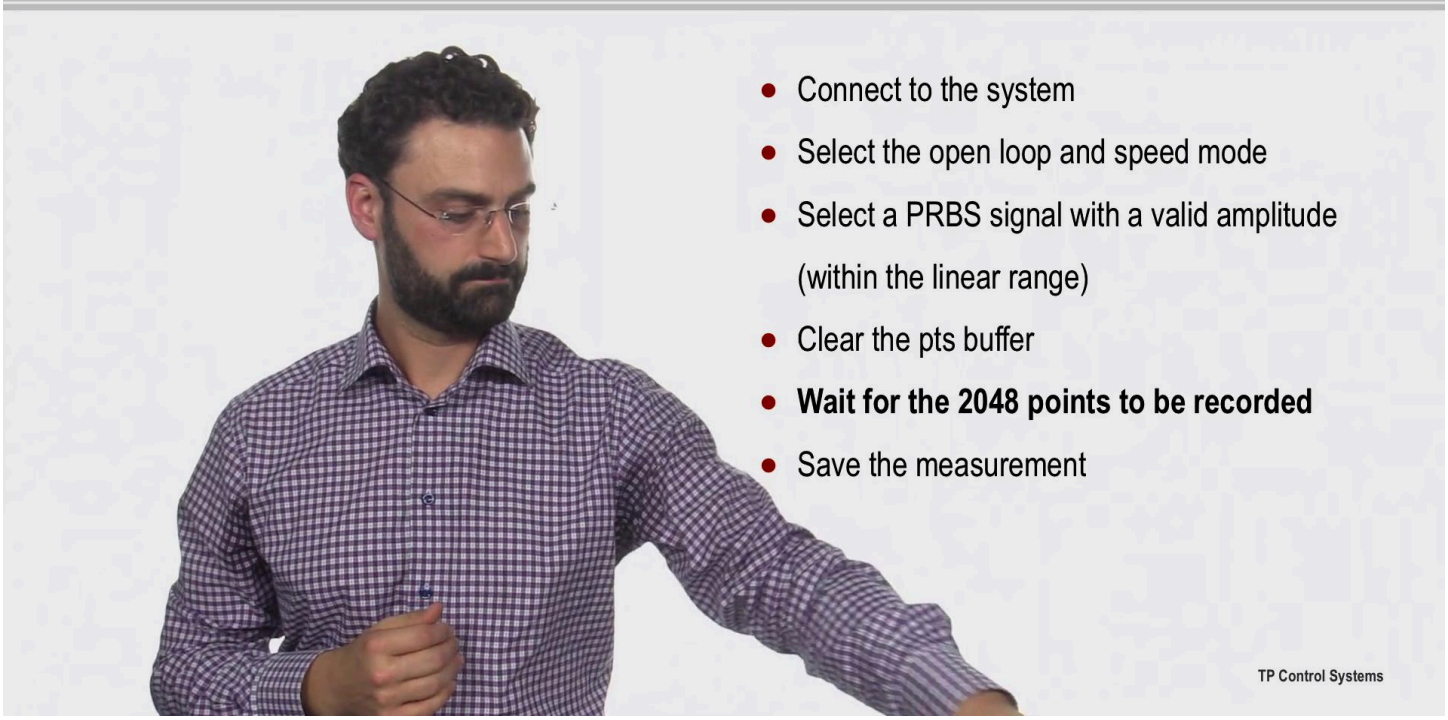
Now, in this experiment, you're not going to compute the empirical transfer function of your system yourself. You're going to use the frequency fitting tool to do this for you. You'll take an experiment, you'll load the data, and then it will compute the empirical transfer function. You can see here, this black line over here is exactly this. This is an empirical transfer function for this particular system.

Notes

Summary

6m 50s

Experiment 1 – PRBS signal



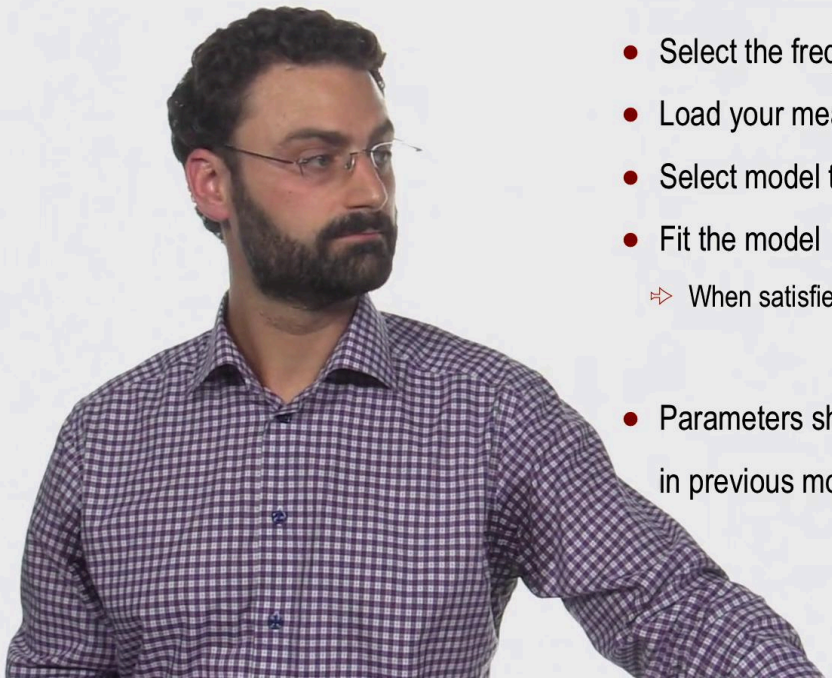
Now in the first experiment, you want to go ahead and connect to the system, select open loop and the speed mode. This means we want to identify the transfer function from the input voltage to the speed of the motor and not to the position. Choose the PRBS signal and pick a valid amplitude. Where valid means that it's within the linear range of your system. That means that the minimum that you pick has to involve the motor moving at least to some small amount. The maximum that you pick has to have the motor not turning so fast that the signal is saturated. That means there has to be a little bit more voltage that you could apply, and the motor still goes a little bit faster. Within that range, the system acts in a linear fashion. If you're outside of that range, then using a linear system identification tool, as we're doing now, simply won't work. Now, in order to estimate an empirical frequency response, you need as much data as you possibly can. Go ahead and clear the buffer and then wait until you've recorded the full buffer length of valid data. In this case, it's 2,048 points. Save the measurement.

Notes

Summary

7m 13s

Experiment 2 – PRBS frequency fit



- Select the frequency-fit tool
- Load your measurement
- Select model to fit
- Fit the model
  - ➪ When satisfied, save parameters
- Parameters should be the same as those found in previous modules!

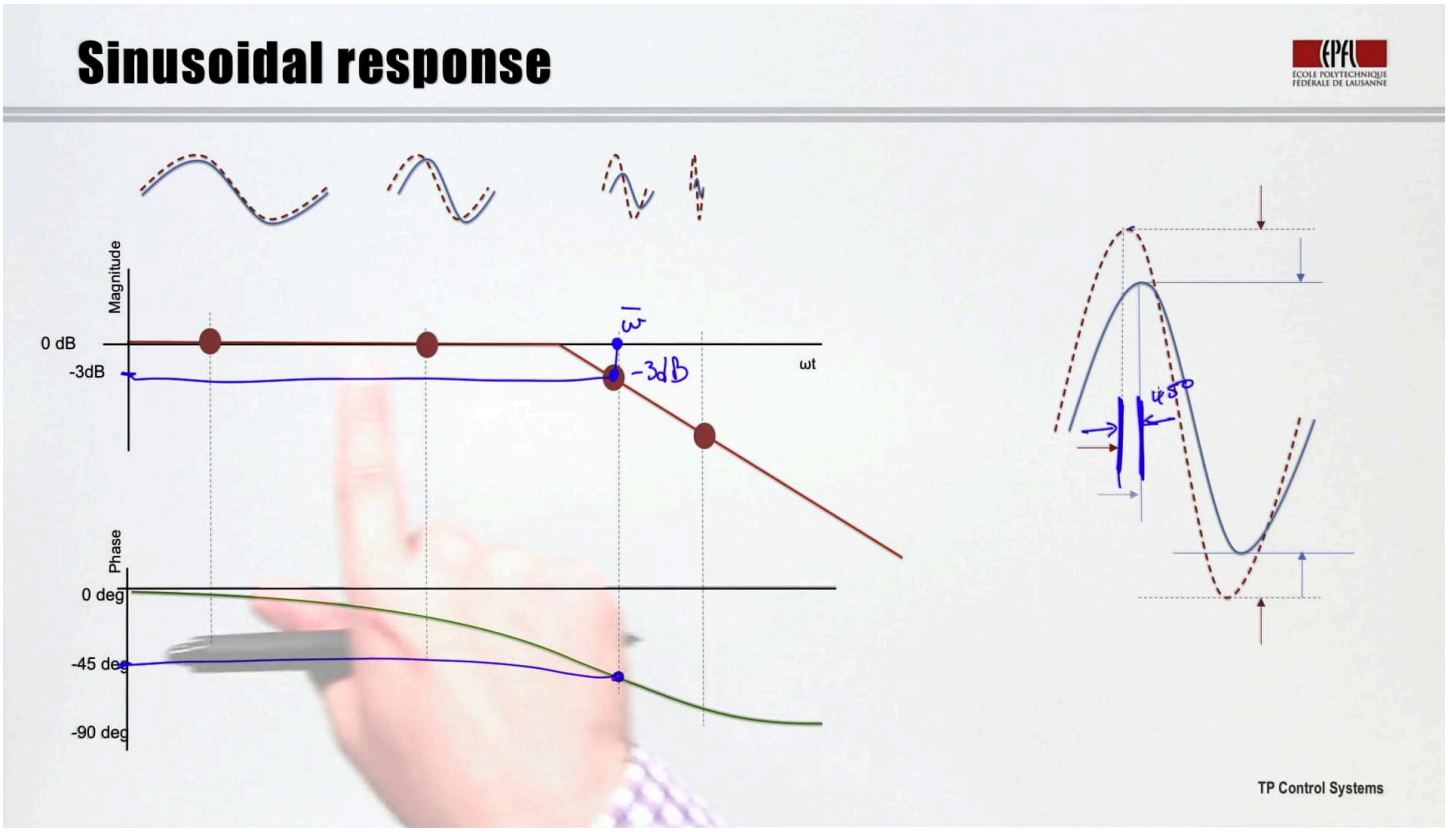
TP Control Systems

The next step is to open the frequency fitting tool and load up the measurement data that you've just taken. Pick an appropriate model to fit to this particular data. You should be able to do that just by glancing at the empirical frequency response that you see. You should be able to approximate or estimate what model class to take. Either that or you can go back to the experiment we did last time where you know from that experiment what the correct model structure should be. Then go ahead and fit the model. It's an interactive system. Choose the right parameters so that you can fit the parameterized model that you're fitting as closely as possible to the empirical data that you've seen. Now, at the end of the day, the parameters that you fit should be exactly the same as the parameters that you saw in the last experiment where we did the time domain fitting. This just simply makes sense. You're identifying the same system using the same model, you should get the same result. Even though we're now using a completely different technique to do that.

Notes

Summary

8m 15s

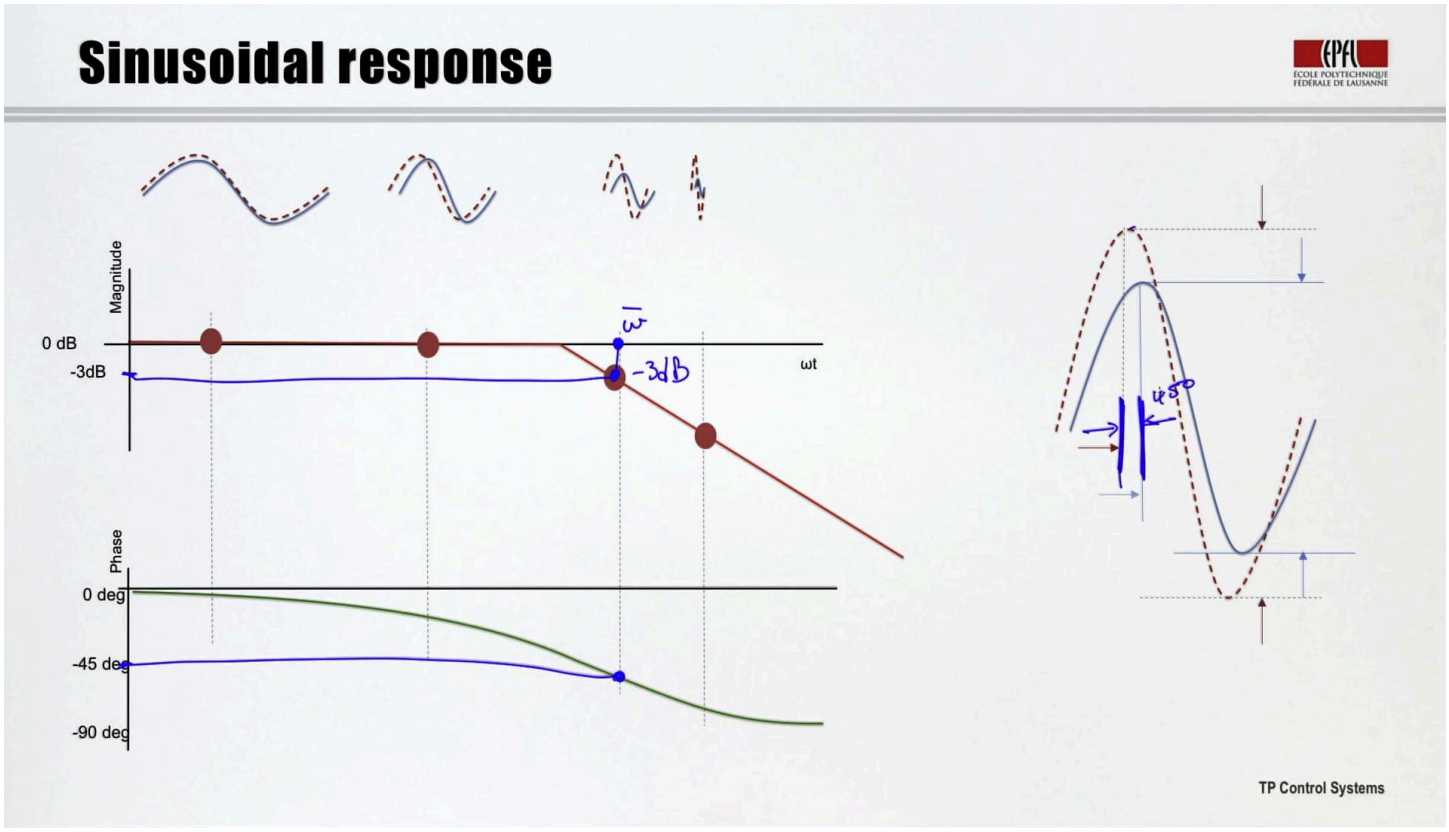


In the second half of the experiment, our goal is to gain an intuitive understanding of what a bode plot means. For example, if I give you this bode plot that you see here and you apply a Sinusoid at, for example, this frequency here, Omega bar, this tells us that we should see an attenuation of about minus three decibels, for example. What that means is that if I apply as an input this red sinusoidal signal and I wait until the system arrives at steady state, the output should also be a sinusoid. The blue signal here of exactly the frequency Omega bar, but that the ratio between the magnitude of my input and the magnitude of my output should be different by three decibels. [inaudible 00:09:57] the blue one should be exactly half the size of the red one. That's exactly what the magnitude plot of a bode plot means. We also want to look at the phase plot. Down here this particular frequency, if we see -45 degrees over here, when we look at the output and the input signals, we should see that the phase of the input signal versus the phase of the output signal should differ by exactly 45 degrees. This is what the bode plot is. What we want you to do is choose four different frequencies.

Notes

Summary

9m 12s



Two down here in the zero decibels per decade range. Before the bandwidth or before the cutoff frequency of your system and two down here where the system is rolling off in the -20 decibels per decade read range.

Notes

Summary

10m 30s

Experiment 3 – Sinusoidal excitation

- Select 4 frequencies
  - 2 in the 0dB/dec and 2 in the -20dB/dec
- Connect to the system
- Select the **open loop** and **speed** mode
- Select the sinusoidal signal with a valid amplitude (within the linear range)
- Apply the 4 frequencies, one after the other
- Save the measurements

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Go ahead and connect back to the system. Choose open loop and speed mode again. But this time pick a Sinusoidal signal to drive your system with and again a valid amplitude. Again in a linear range. Apply each one of your four frequencies in turn, one after the other, ensuring that you do enough periods of each frequency, so that your system has settled down to a steady state. We're not trying to measure transients here. We want to measure steady-state behaviour.

Notes

Summary

10m 45s

Experiment 4 – Bode plot

- Load your measurement with the temporal-fit tool
- For each of the four frequencies, compute the attenuation and phase
- Using the frequency fit tool, load the PRBS measurement (experiment 2)
- Compare computed values with those obtained via the PRBS signal
- The frequency fit tool uses [rad/sec] while the remote experimentation uses [Hz]. You will have to do the conversion before reporting the points

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In the last experiment, take the data that you've recorded and load it into the time domain tool, into temporal fitting tool. For each of the four frequencies, measure the attenuation or the ratio between your input magnitude and your output magnitude, as well as the relative phase between your input and your output. Record those and then go back to the frequency fitting tool and load up the PRBS data that you had previously. What we want you to do is to go to each of the frequencies of the four sinusoids that you applied and confirm that the attenuation and the phase shift that you measured directly match the empirically identified magnitude and phase shift of the bode plot that you saw previously. Of course, these should match exactly. If they don't, it's almost certainly because you're outside of your linear range. Now, one minor detail is that the frequency fitting tool is measured in radians per second, whereas the remote experiment uses hertz. Before you record your results, make sure you do the quick conversion.

Notes

Summary

11m 11s

Conclusions

What you have learned today

- Non parametric model can be obtained experimentally using its frequency response

1. PRBS signals excite many frequencies at the same time
2. Frequency response can be measured via recording sinusoids at multiple frequencies

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What we saw today is that you can fit or you can learn a non-parametric model experimentally by focusing on the frequency response of your system. We saw two different ways of doing this. The first is that we excited the system with an input signal that excites a wide range of different frequencies. In this case, we've used a PRBS signal, but there's a range of other different types of signals that we might use as well. Things like chirp signals or frequency sweep signals, depending on the system that we're trying to identify. They all have the same goal. They want to excite the whole range of output frequencies that we're interested in. We can then use this input-output data in order to build an empirical transfer function model. The second approach that we took was to directly apply sinusoidal input, wait until it arrives at steady state, and then measure the attenuation and the phase shift of the system. This allows us then to measure the bode plot at very specific points at very specific different frequencies. But this, for the most part, is a bit of an academic exercise. The purpose of doing this was really just to gain an intuition or a feeling of what a bode plot means in an experimental setting. We generally would not use this, the second approach. In order to fit a model of a system, we would use something much closer to the first approach.

Notes

Summary

12m 10s